LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRDSEMESTER – APRIL 2018

16PMT3MC01- TOPOLOGY

Date: 30-04-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer all the questions. Each question carries 20 marks.

I.a)1) Define the following in a topological space: (i) isolated point (ii) interior point and (iii) boundary point.

OR

a)2) In any metric space prove that each open sphere is an open set. (5)

b)1) State and prove Cantor's intersection theorem.

b)2) Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous $\Leftrightarrow f^{-1}(G)$ is open whenever G is open in Y. (7+8)

OR

c)1) Let X be a metric space and let Y be a complete metric space, and let A be a dense subspace of X. If f is uniformly continuous mapping of A into Y then prove that f can be extended uniquely to a uniformly continuous mapping g of X into Y.

c)2) State Cauchy and Minkowski's inequalities. (10+5)

II.a)1) Prove that every separable metric space is second countable.

OR

a)2) Define finite intersection property and state the equivalent form of the theorem: A topological space is compact iff every class of closed sets with empty intersection has a finite subclass with empty intersection. (5)

b)1) State and prove Lindelof's theorem

b)2) State and prove Tychonoff's theorem. (6+9)

OR

c)1) State and prove Heine Borel theorem.

c)2) Define Product topology and explain with an example. (10+5)

III.a)1) Quoting the necessary results prove that a metric space is compact iff it is complete and totally bounded.

OR

a)2) Quoting the necessary results prove that every sequentially compact metric space is compact. (5)

b) State and prove Ascoli's theorem. (15)

OR

c)1) Prove that a metric space is sequentially compact iff it has the Bolzano Weierstrass property.

c)2) State and prove Lebesgue's covering lemma. (7+8)

IV. a)1) Prove that the range of a continuous real function defined on a connected space is an interval.

OR

a)2) Prove that a topological space X is disconnected iff there exists a continuous mapping of X onto the discrete two- point space {0,1}. (5)

b)1) Let X be a normal space and let A and B be disjoint closed subspace of X. If [a,b] is any closed interval on the real line, then prove that there exists a continuous real function f defined on X, all of whose values lie in [a.b] such that f(A) = a and f(B) = b.

b)2) Stating the results prove that R^n and C^n are connected. (7+8)

OR

c) State and prove Urysohn's lemma. (15)

V. a)1) Prove that X_∞ is Hausdorff.

OR

a)2) Explain the advantage of one point compactification to loacally compact Hausdorff spaces. (5)

b) State and prove Weierstrass approximation theorem.

OR

c) State Complex Stone Weierstrass theorem. Prove the two lemmas that are required to prove its Real case. (15)

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