



**LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**

**M.Sc. DEGREE EXAMINATION – MATHEMATICS**

THIRD SEMESTER – APRIL 2018

**16PMT3MC04- ALGORITHMIC GRAPH THEORY**

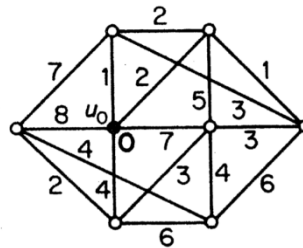
Date: 04-05-2018  
Time: 09:00-12:00

Dept. No.

Max. : 100 Marks

**ANSWER ALL QUESTIONS**

- I a) Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group.  
[OR]
- b) Show that if every vertex of a graph  $G$  has degree 2, then  $G$  contains a cycle. (5)
- c) i) State and prove Chavatal theorem for hamiltonian graphs.  
ii) Prove that a graph is bipartite if and only if it contains no odd cycle. (10+ 5)  
[OR]
- d) (i) With usual notations prove that  $\alpha^1(G) + \beta^1(G) = n(G)$  .  
(ii) Apply Dijkstra's algorithm to find shortest path from  $u$  to all other vertices of the following graph.



(5+ 10)

- II a) Write about travelling salesman problem.  
[OR]
- b) Show that a connected graph has an Euler's trail if and only if it has atmost two vertices of odd degree . (5)
- c) i) Show that closure of a graph is well defined  
ii) Let  $G$  be a simple graph with  $v \geq 3$ . Prove that if  $c(G)$  is complete then  $G$  is Hamiltonian. (7+8)  
[OR]
- d) i) State and prove Dirac theorem for hamiltonian graphs.  
ii) Write about Chinese postman problem. (10+5)
- III a) Show that a tree has atmost one perfect matching.  
[OR]
- b) Find the number of different perfect matchings in  $K_{2n}$  . (5)
- c) i) Prove that a matching  $M$  in  $G$  is a maximum matching iff  $G$  contains no  $M$  – augmenting path.  
ii) Let  $M$  be a matching and  $K$  be a covering with  $|M|=|K|$ . Then show that  $M$  is a maximum matching and  $K$  is a minimum covering. (10+5)

[OR]

- d) i) Define split graph and prove that complement of a split graph is a split graph.  
ii) State and prove any three properties of a split graph.

(8 + 7)

IV a) Write BFS algorithm.

[OR]

b) Define permutation graph and give an example. (5)

c) i) State and prove the characterization theorem for split graphs.

ii) State and prove the characterization theorem for triangular graphs. (10 + 5)

[OR]

d) i) Write DFS algorithm and discuss about its efficiency .

ii) Discuss about transitive orientation property. (10+ 5)

V a) Define interval graph and give an example.

[OR]

b) Define permutation labeling with an example. (5)

c) i ) State and prove Tucker's theorem for circular-arc graphs.

ii) Prove that every interval graph is a circular-arc graph. (10 + 5)

[OR]

d) i) Discuss any three applications of interval graphs.

ii) State and prove the characterization theorem for interval graphs. (6 + 9)

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