LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FOURTHSEMESTER – APRIL 2018

16PMT4MC04– CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

Date: 23-04-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer ALL Questions:

1. a) Write short note on Iterated kernel and reciprocal kernel. (5)

OR

b) Show that the function $y(x) = xe^x$ is a solution of the volterra integral equation $y(x) = sinx + 2\int_0^x \cos(x-t)\, y(t)dt$. (5)

c) Derive the general solution of homogeneous Fredholm integral equation of the second kind with separable kernel. (15)

OR

d) Determine the eigen values and eigen function of the homogeneous integral equation
$y(x) = \lambda \int_0^1 \mathrm{K}(x+t)\, y(t)dt$, where $K(x,t) \begin{cases} t(x+1), 0 \le x \le t, \\ x(t+1), t \le x \le 1. \end{cases}$ (15)

2. a) Solve $y(x) = cosx + \lambda \int_0^{\pi} sinxy(t)dt$ using Fredholm integral equation of the second kind (5)

OR

b) Invert the integral equation $y(x) = f(x) + \lambda \int_0^{2\pi} (sinxcost)y(t)dt$. (5)

c) Show that the integral equation $y(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t)\, y(t)dt$ possesses no solution for $f(x) = x$, but that it possesses infinitely many solutions when
$f(x) = 1$. (15)

OR

d) State and prove Fredholm alternative theorem. (15)

d. a) Find the resolvent kernel of the Volterra integral equation with the kernel
$k(x,t) = 1$ (5)

OR

b) Find the resolvent kernels for the Fredholm integral equation
$k(x,t) = (1+x)(1-t), a = -1, b = 1.$ (5)

c) State and prove Hilbert-Schmidt theorem. (15)

OR

d) Find the solution of Fredholm integral equation of the second kind by (i) successive approximation (ii) Iterative method and (iii) Neumann series. (15)

d. a) Write a short note on proximity of curves.

OR

b) Find the extremum of the function $\int_{x_0}^{x_1} \frac{\left(1+(y')^2\right)^{\frac{1}{2}}}{x} dx$. (5)

c) (i) State and prove Euler's equation.

(ii) If the functional $I[y(x)]$ attains a maximum or minimum on $y = y_0(x)$, where the functional belongs to a certain class, then show that at $y = y_0(x)$, $\delta I = 0$.(10+5)

OR

d) (i) Test for an extremum the functional $I[y(x)] = \int_0^1 (xy + y^2 - 2y^2 y')dx$,

$y(0) = 1, y(1) = 2$.

(ii) Derive the variation problem of parametric form. (5+10)

5. a) Derive Field of Extremals (5)

OR

b) Give a brief writing on Legendre Condition. (5)

c) Derive the Transversality condition and Find the shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$. (15)

OR

d) Explain in detail the variation problem with a movable boundary for a functional dependent on two functions. (15)

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