LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FOURTHSEMESTER – APRIL 2018

16UMT4ES01- COMBINATORICS

Date: 23-04-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART A

Answer ALL the questions. **(10 X 2 = 20)**

1. If $a_n = na_{n-1}$ and $a_1 = 1$, find a_n.
2. Define a generating function.
3. There are 5 seats in a row available, but 12 people to choose from. How many different seating are possible?
4. Expand $(1+x)^8$.
5. If $A_1 = \{1,2\}$, $A_2 = \{4\}$, $A_3 = \{1,3\}$, and $A_4 = \{2,3,4\}$, find the distinct representatives for the sets A_i.
6. Construct 2 different 5 x 5 Latin squares which have the same first rows, but no other rows the same.
7. Define a tree.
8. Write down all the possible derangements of 1234.
9. 6 men are to be seated round a circular table. How many ways are there of achieving this?
10. Define the inclusion and exclusion principle.

PART B

Answer any FIVE questions. **(5 X 8 = 40)**

11. Suppose it is known that $t(n,n-1) = 1$ and $(n-k-1)\,t(n,k) = k(n-1)\,t(n,k+1)$ for each $k < n-1$. Prove that $t(n,k) = \frac{(n-1)^{n-k-1}(n-2)!}{(k-1)!\,(n-k-1)!}$.
12. Prove that if a graph has $2n$ vertices, each of degree $\geq n$, then the graph has a perfect matching.
13. State and prove the Landau's theorem.
14. Find a_n if $a_n = 4a_{n-1} + 4a_{n-2} - 16a_{n-3}$, $a_1 = 8$, $a_2 = 4$, $a_3 = 24$.
15. Let S be a set of mn objects. Prove that S can be split up into n sets of m elements in $\frac{(mn)!}{m!^n\,n!}$ different ways.
16. Explain Ordered Selection and evaluate the following: (a) $p(7,4)$, (b) $p(9,5)$.
17. (a) How many permutations are there of the 26 letters of the alphabet in which the 5 vowels are in consecutive places.

    (b) How many different necklaces can be designed from n colors, using one bead of each color?
18. Find the Rooks polynomial for an ordinary 4×4 board.

PART C

Answer any TWO questions. **(2 X 20 = 40)**

19. (a) Suppose that each of k indistinguishable golf balls has to be coloured with any one of n given colours. Use recurrence relation approach, to find the number of different colourings and hence deduce when $n = 4$ and $k = 2$.

(b) State and prove the marriage theorem. (10+10)

20. (a) Find the optimal assignment for the following problem.

		Man			
Job		A	B	C	D
	a	6	8	2	7
	b	5	8	13	9
	c	2	7	8	9
	d	4	11	7	10

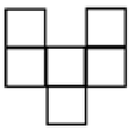
(b) Solve the Fibonacci-type relations. (10+10)

21. (a) n-digit integer sequences are to be formed using only the integers 0, 1, 2, 3. (i) How many n-digit sequences are there? (ii) How many n-digit sequences have an odd number of 0's?

(b) State and prove the exchange property. (10+10)

22. (a) Let n be a positive integer. Show that if $(1 + x)^n$ is expanded as a sum of powers of n, the coefficient of x^r is $\binom{n}{r}$.

(b) Find the rook polynomial of the board



(10+10)

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