# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2018**

**16UMT4MC01– ABSTRACT ALGEBRA**

Date: 20-04-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

## SECTION – A

**Answer ALL questions (10 × 2 = 20)**

1. If $\sigma : S \to T$ and $\phi : T \to U$ are one – to – one mappings, show that $\phi \circ \sigma$ is also is an one – to – one mapping.
2. Show that if every element of a group G is its own inverse, then G is abelian.
3. Show that the intersection of two normal subgroups of G is also a normal subgroup of G.
4. Define quotient group.
5. Let G be the group of all integers with operation addition. Is the mapping $h: G \to G$ defined by $h(x) = x+1$ for all x in G, a group homomorphism?
6. Define inner automorphism of a group.
7. When do you say that an integral domain is of finite characteristic?
8. Show that kernel of a ring homomorphism is an ideal.
9. Find all units of J[i].
10. Give an example of a Euclidean ring.

## SECTION – B

**Answer any FIVE questions (5 × 8 = 40)**

11. If G is group of even order, prove that it has an element $a \neq e$ satisfying $a^2 = e$.
12. Show that any group of prime order is cyclic.
13. Show that a subgroup N of a group G is a normal subgroup of G if and only if the product of two right cosets of N in G is again a right coset of N in G.
14. If G is a group, show that the set of all automorphisms $\mathcal{A}(G)$ of G is also a group.
15. Show that the alternating group $A_n$ is a normal subgroup of the symmetric group $S_n$ of index two.
16. State and prove Cayley's theorem.

17. Let R be the ring of all real valued continuous functions on [0,1].Show that M={f(x) ε R : f($\frac{1}{2}$)= 0 } is a maximal ideal of R.

18. Let R be a Euclidean ring. Show that any two elements a and b in R have a greatest common divisor d and d = λa + μb for μ,λ in R.

## SECTION – C

**Answer any TWO questions (2 × 20 = 40)**

19 (a) Show that any positive integer a >1 can be factored in a unique way as product of prime numbers.

(b) If H and K are finite subgroups of a group G, show that o(HK) = $\frac{o(H)o(K)}{o(H\cap K)}$.

20 (a) If φ is homomorphism of a group G onto G', N' is a normal subgroup of G' and N= { xε G: φ(x)ε N'}, show that G/N ≈ G'/N'.

(b) State and prove Lagrange's theorem.

21 (a) Prove that any field is an integral domain.

(b) If U is an ideal of a ring R show that R/U is also a ring and is a homeomorphic image of R.

22 (a) Let A be an ideal of a Euclidean ring R. Show that there exists an element $a_0$ in A such that A consists exactly of all $a_0x$ as x ranges over R.

(b) Show that J[i] is a Euclidean ring.

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