LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRSTSEMESTER – APRIL 2018

17/16PMT1MC05- PROBABILITY THEORY AND STOCHASTIC PROCESS

Date: 02-05-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

Answer ALL the questions. Each question carries equal marks.

1. (a) A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find k, (ii) Evaluate $P(X<6), P(X\geq 6)$, and $P(0<X<5)$ (iii) If $P(X\leq a)>\frac{1}{2}$, find the minimum value of a.

OR

(b) With usual notations prove that $-1\leq \gamma_{XY}\leq 1$. (5)

(c) Let (X,Y) be a two dimensional random variable uniformly distributed over the triangular region bounded by $y=0, x=3$ and $y=\frac{4}{3}x$. Obtain the correlation coefficient between X and Y.

OR

(d) Obtain the Correlation coefficient for the following data:

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

(15)

2. (a) State and prove Chebychev's inequality.

OR

(b) Two unbiased dice are thrown. If X is the sum of the numbers showing up, prove that $P(|X-7|\geq 3)\leq\frac{35}{54}$. Compare this with the actual probability. (5)

(c) State and prove the necessary and sufficient condition for the weak law of large numbers.

OR

(d) State and prove two Borel-Cantelli Lemmas. (15)

3. (a) State and prove invariance property of Consistent Estimators.

OR

(b) If T_1 and T_2 are two unbiased estimators of $\gamma(\theta)$, having the same variance and ρ is the correlation between them, then show that $\rho\geq 2e-1$, where e is the efficiency of each estimator.

(5)

(c) A minimum variance unbiased (M. V. U) estimator is unique in the sense that if T_1 and T_2 are M. V. U estimators for $\gamma(\theta)$ then prove that $T_1=T_2$, almost surely. (15)

OR

(d) (i) If a sufficient estimator exists then prove that it is a function of Maximum Likelihood Estimator.

(ii) Find the maximum likelihood estimate for the parameter λ of a Poisson distribution on the basis of a sample of size n. Also find its variance. (8 + 7)

4. (a) Explain Critical region and two types of Errors.

OR

(b) If $x \geq 1$, is the critical region for testing $H_0: \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population $f(x, \theta) = \theta e^{-\theta x}, 0 \leq x < \infty$, obtain the values of type I and type II errors. (5)

(c) State and prove Neyman-Pearson Lemma.

OR

(d) Given the frequency function $f(x, \theta) = \begin{cases} \frac{1}{\theta} & 0 < x < \infty, \theta > 0 \\ 0 & elsewhere \end{cases}$ and what you are testing the null hypothesis $H_0: \theta = 1$ against $H_1: \theta = 2$, by means of a single observed value of x, what would be the sizes of the type I and type II errors, if you choose the interval (i) $0.5 \leq x$, (ii) $1 \leq x \leq 1.5$ as the critical region? Also obtain the power function of the test.

(e) Write down the advantages and drawbacks of non-parametric tests. (8 + 7)

5. (a) Write a short note on classification of stochastic process.

OR

(b) Describe the procedure used in median test. (5)

(c) Prove that a homogeneous Markov chain $\{X_n\}$ satisfies the relation $P_{ij}^{(n+m)} = \sum_k P_{ik}^{(n)} P_{kj}^{(m)}$ for every $n, m \geq 0$ provided we define $p_{ij}^{(0)} = \delta_{ij}$, where $\delta_{ij} = \begin{cases} 1 & if\, i = j \\ 0 & if\, i \neq j \end{cases}$.

OR

(e) Let $\boldsymbol{P}$ be the transition probability matrix of a homogeneous finite Markov chain with elements $p_{ij}(i, j = 0,1,2, \dots, k-1)$. Then prove that the n-step transition probabilities $p_{ij}^{(n)}$ are obtained as the elements of the matrix $\boldsymbol{P}^n$. (15)

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