LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – **STATISTICS**

FIRSTSEMESTER – APRIL 2018

17/16UMT1AL02- MATHEMATICS FOR STATISTICS - I

Date: 30-04-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

SECTION A

ANSWER ALL QUESTIONS. (10 × 2 = 20)

1. Define upper triangular matrix, give an example.
2. Write any two properties of determinants.
3. Find the Eigen value of the matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$.
4. Find the matrix X from the given equation: $\begin{bmatrix} 4 & -3 \\ 5 & 2 \end{bmatrix} - 3X = \begin{bmatrix} -3 & 2 \\ 5 & 1 \end{bmatrix}$
5. If $y = x^3 - 3logx + e^x + 2tanx$, then find $\frac{dy}{dx}$.
6. Find $\frac{d}{dx}\left(\frac{logx}{x^2}\right)$.
7. Find the n^{th}derivative of $y = e^{ax}$.
8. Write the conditions for concave upward.
9. Prove that $\int_0^{\frac{\pi}{2}} sin^n x dx = \int_0^{\frac{\pi}{2}} cos^n x dx$.
10. Evaluate:$\int \frac{dx}{1+9x^2}$.

SECTION B

ANSWER ANY FIVE QUESTIONS. (5 × 8 = 40)

11. Solve the following simultaneous equations: $x + y + 3z = 5,\ 2x + y + z = 4,\ x + 2y + 5z = 8$.
12. Show that the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -3 \\ 5 & 4 & -4 \end{bmatrix}$ satisfies the equation $A(A - I)(A + 2I) = 0$.
13. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$and $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$, Verify that $(AB)^t = B^t A^t$, where A^t is the transpose of A.
14. Find the maximum value of$\frac{\log x}{x}$ for positive values of$x$.
15. If $y = x^{x^{x^{\dots to \infty}}}$, then find $\frac{dy}{dx}$.
16. Find the n^{th}derivative of $y = \frac{x^2}{(x-1)^2(x+2)}$
17. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{asinx + bcosx}{sinx + cosx} dx$.
18. Prove that $\int_0^{\frac{\pi}{4}} \log(1 + tan\theta)\, d\theta = \frac{\pi}{8} \log 2$.

SECTION C

ANSWER ANY TWO QUESTIONS. (2 x 20 = 40)

19. (a) Verify Cayley- Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 & -3 \\ 2 & 3 & -1 \\ 2 & 3 & 0 \end{bmatrix}$.

(b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 8 & 2 \\ 4 & 9 & -1 \end{bmatrix}$. **(14+6)**

20. (a) (i) Without using logarithm find the derivative of $y = \frac{e^x + sinx}{secx - x^5}$

(ii) Using logarithm differentiate $y = (\tan x)^{\log x}$. **(5+5)**

(b) Find the points of inflection of the cubic curve $y = \frac{a^2 x}{x^2 + a^2}$ and show that they lie on a straight line. **(10)**

21. (a) If $y = a\cos(logx) + bsin(logx)$ then show that

$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

(b) Differentiate $e^{sin^{-1}x}$ with respect to $sin^{-1}x$. **(14+6)**

22. (a) Evaluate: $\int \frac{x}{\sqrt{x^2+x+1}} dx$.

(b) Evaluate: $\int \frac{3x+1}{(x-1)^2(x+3)} dx$. **(10+10)**

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