

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – APRIL 2018

17/16UMT1MC01– ALGEBRA AND CALCULUS - I

Date: 25-04-2018

Dept. No.

Max. : 100 Marks

Time: 01:00-04:00

PART – A

Answer all questions:

(10 X 2 = 20)

1. Find the n^{th} derivative of $\sin x$.
2. At any point on the curve $y = b \sin \frac{x}{a}$ find the lengths of the sub tangent and sub normal.
3. Write the condition for the maxima and minima of functions of two variables.
4. State the condition used in Lagrange's method of undetermined multipliers.
5. Find the radius of curvature of the curve $xy = 30$ at the point $(3,10)$.
6. Define evolute of the curve.
7. Find the product of the roots of the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$.
8. Form a quadratic equation having $-1 + \sqrt{-1}$ as a root.
9. If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, find the value of $\alpha^2 + \beta^2 + \gamma^2$.
10. State the condition for the roots of the cubic equation to be in harmonic progression.

PART – B

Answer any five questions:

(5 X 8 = 40)

11. Find y_n when $y = \frac{x^2}{(x-1)^2(x+2)}$.
12. Find the angle of intersection of the cardioids $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$
13. Investigate the maximum and minimum values of the function $f(x, y) = 4x^2 + 6xy + 9y^2 - 8x - 24y + 4$.
14. Find the radius of curvature of the cardioid $r = a(1 - \cos \theta)$
15. Find the asymptotes of the equation $y^3 - 6xy^2 + 11x^2y - 6x^3 + x + y = 0$.
16. Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$ given that $1 + \sqrt{-1}$ is a root of it.
17. Show that the equation $x^6 + 3x^2 - 5x + 1 = 0$ has at least four imaginary roots.
18. Solve the equation $x^3 - 6x - 9 = 0$ using Cardon's method.

PART – C

Answer any two questions:

(2 X 20 = 40)

19. (a) If $y = (x + \sqrt{1+x^2})^m$, prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$

(b) Find the slope of the tangent with the initial line for the cardioids $r = a(1 - \cos \theta)$ at $\theta = \frac{\pi}{6}$.

(15+5)

20. Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

21. (a) Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of its roots are equal in magnitude and opposite in sign.

(b) Solve the equation $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$. (10+10)

22. (a) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in geometric progression.

(b) Using Horner's method show that the equation $x^3 - 6x - 13 = 0$ has one real root between 3 and 4. Find it to two places of decimals. (6+14)
