LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

SECONDSEMESTER – APRIL 2018

MT 2812- PARTIAL DIFFERENTIAL EQUATIONS

Date: 21-04-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Answer ALL Questions

I. a) (i)) Solve $Z^2 = pqxy$

OR

(ii) Solve $p^2 q (x^2 + y^2) = p^2 + q$ (5 Marks)

b) (i) Find the characteristic equation of $z = p^2 – q^2$ and determine the integral surface which passes through the parabola $4z + x^2 = 0, y = 0$. (15 Marks)

OR

(ii) Obtain the condition for compatibility of f(x, y, z, p, q) = 0 and g(x, y, z, p, q) = 0 (8 Marks)

(iii) Show that $xp – yq = x$ and $x^2 p + q = xz$ are compatible and find the solution. (7Marks)

II a) (i) Solve $(D^2 – 2 D D' + D'^2) z = x^3$

OR

(ii)Solve $(D^2 + D'^2) z = \cos 4x.\cos 3y$ (5 Marks)

b) (i) Obtain the canonical form for hyperbolic partial differential equation. (8 Marks)

(ii) Reduce $y^2r –2xys + x^2t = (y^2 /x)p + (x^2 /y) q$ to a canonical form. (7 Marks)

OR

(iii) Prove that the transformation of the independent variable does not modify the type of the Partial Differential Equation. (10 Marks)

(iv) Reduce $3U_{xx} + 10U_{xy} + 3U_{yy} = 0$ to the canonical form and solve. (5 Marks)

III a) (i) Study the Transmission Line problem.

OR

(ii) Derive one-dimensional wave equation. (5 Marks)

b) (i) State and prove the Maximum and Minimum principle of the solution of the heat conduction equation. Show that it is unique

OR

(ii) Obtain the solution of Diffusion equation in spherical coordinates. (15 Marks)

IV. a) (i) Prove that the application of an integral transforms to a partial differential equation reduces the independent variables by one.

OR

(ii) Show that the Green's function has the symmetry property. (5 Marks)

b) (i) Solve the heat conduction equation given by $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, -\infty < x < \infty, t > 0$ subject to the initial and boundary conditions $u(x,t) \to 0$ and $\frac{\partial u}{\partial x}(x,t) \to 0, as |x| \to \infty, u(x,0) = f(x), -\infty < x < \infty$.

OR

(ii)Obtain the solution of the interior Dirichlet's problem for a sphere using the Green's function method. (15 Marks)

V. a) (i) With the help of the resolvent kernel, find the solution of $\emptyset(x) = x + \int_0^x (\xi - x)\, \emptyset(\xi) d\xi$.

OR

(ii) Show that all iterated kernel of a symmetric kernels are also symmetric.(5marks)

b) (i) Find the solution of Volterra's integral equation of second kind by the method of successive substitutions.

OR

(ii) State and prove Hilbert- Schmidt theorem. (15 Marks)

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