LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION –**MATHEMATICS**

SECOND SEMESTER – **APRIL 2018**

MT 2502– ALGEBRA AND CALCULUS - II

Date: 27-04-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

Answer ALL questions. **(10 × 2 = 20)**

1. If $f(x)$ is an odd function show that $\int_{-a}^{a} f(x)dx = 0$.
2. Compute $\int xe^x \ dx$.
3. Evaluate $\int_0^1 \int_0^1 (x^2 + y^2)dx\, dy$.
4. If x + y = u and y = uv, find $\frac{\partial(x,y)}{\partial(u,v)}$.
5. Show that $\Gamma(n + 1) = n!$ if n is a positive integer.
6. Using Beta function evaluate $\int_0^1 x^7(1 - x)^8 dx$.
7. Show that 1 + $\frac{1}{1!}$ + $\frac{1}{2!}$ + $\frac{1}{3!}$ + ………. is convergent.
8. State Cauchy root test for convergence.
9. Write down the expansion of $(1 + x)^{-p/q}$.
10. Find the coefficient of x^n in the expansion of 1 + $\frac{1+2x}{1!}$ + $\frac{(1+2x)^2}{2!}$ + ...

PART – B

Answer any FIVE questions. **(5 × 8 = 40)**

11. Find the area of the surface generated by revolving the arc of the catenary y = cosh (x/c) from x = 0 to x = c about the x-axis.
12. By changing the order of integration, evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$.
13. Evaluate $\iint (x^2 + y^2)\, dx\, dy$ over the region for which x, y $\geq$ 0 and x+y ≤1.
14. Using Gamma functions evaluate $\int_0^1 x^m \,(\log\frac{1}{x})^n\, dx$.
15. Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots\ldots\ldots.$
16. Discuss the convergence of the series $\frac{1}{1+x} + \frac{1}{1+2x^2} + \frac{1}{1+3x^3} + \cdots$.

17. Sum the series $\frac{1^2}{3!}+\frac{2^2}{5!}+\frac{3^2}{7!}+$ ....

18. Sum the series $\sum_{n=1}^{\infty}\frac{1}{n\,(n+1)}\frac{1}{3^n}$.

PART – C

Answer any TWO questions. **(2 × 20 = 40)**

19 (a) Evaluate $\int_0^{\pi/2}\frac{a\sin x+b\cos x}{\sin x+\cos x}\,dx$.

(b) Obtain the reduction formulae for $I_{m,n}=\int x^m(\log x)^n\,dx$ where n and m are positive integers. Hence find the value of $\int x^4(\log x)^3\,dx$. **(10+10)**

20 (a) Evaluate $\iint_R xy\,dx\,dy$ where R is the region in the first quadrant bounded by the hyperbolas $x^2 - y^2 = a^2$ and $x^2 - y^2 = b^2$ and the circles $x^2 + y^2 = c^2$ and $x^2 + y^2 = d^2$ $(0<a<b<c<d)$.

(b) By changing the order of integration, evaluate $\int_0^a\int_{\frac{x^2}{a}}^{2a-x} xy\;dx\,dy$.**(10+10)**

21 (a) Show that $\beta\,(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

(b) Show that $\Gamma(\frac{1}{2})=\sqrt{\pi}$. **(16+4)**

22 (a) Show that series $\frac{1}{1^k}+\frac{1}{2^k}+\frac{1}{3^k}+$ .... is convergent if k >1 and dinvergent if k=1.

(b) Show that $\log\sqrt{12}=1+(\frac{1}{2}+\frac{1}{3})\frac{1}{4}+(\frac{1}{4}+\frac{1}{5})\frac{1}{4^2}+(\frac{1}{6}+\frac{1}{7})\frac{1}{4^3}+\ldots\ldots$ **. (10+10)**

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