LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

SECONDSEMESTER – APRIL 2018

MT 2811- MEASURE THEORY AND INTEGRATION

Date: 19-04-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer **ALL** questions:

1. (a) For any sequence of sets $\{E_i\}$, prove that $m^*(\bigcup_{i=1}^{\infty} E_i) \leq \sum_{i=1}^{\infty} m^*(E_i)$. **(5)**

(OR)

(b) If f and g be real valued measurable functions defined on a set E, then prove that $f - g$ and fg are also measurable . **(5)**

(c)Prove that the outer measure of an interval equals to its length. **(15)**

(OR)

(d) (i) Prove that not every measurable set is a Borel set.
(ii)Prove that there exists a non-measurable set. **(7+8)**

2. (a)) Let A and B be any two disjoint measurable sets. If ϕ is a simple function then prove that (i) $\int_{A\cup B} \emptyset dx = \int_A \emptyset dx + \int_B \emptyset dx$(ii) $\int a\emptyset dx = a\int \emptyset dx$, if a > 0. **(5)**

(OR)

(b) Show that $\int_0^{\infty} \frac{sint}{e^t - x} dt = \sum_{n=1}^{\infty} \frac{x^{n-1}}{n^2+1}, -1 \leq x \leq 1.$ **(5)**

(c) State and prove Fatou's lemma. **(15)**

(OR)

(d) (i) State and prove Lebesgue's Dominated Convergence Theorem.
(ii)If f is Riemann integrable and bounded over the finite interval $[a, b]$, then prove that f is integrable $R\int_a^b f dx = \int_a^b f dx$. **(7+8)**

3. (a) Show that if μ is a σ-finite measure on a ring $\mathfrak{R}$, then prove that the extension $\bar{\mu}$ of μ to S^* is also σ-finite. **(5)**

(OR)

(b) Show that every algebra is a ring and every σ-algebra is a σ-ring. **(5)**

(c)If μ is a σ-finite measure on a ring $\mathfrak{R}$, then prove that it has a unique extension to the σ-ring $S(\mathfrak{R})$. **(15)**

(OR)

(d) Let μ^* be an outer measure on $\mathcal{H}(\mathfrak{R})$ and let S^* denote the class of μ^* -measurable sets. Then prove that S^* is a σ –ring and μ^* restricted to S^* is a Complete Measure. **(15)**

4. (a) Let ψ be strictly convex, then prove that ψ(∫ fdμ) $=$ ∫ ψ • f $d\mu$ if and only if $f =$ ∫ f dμ a.e. **(5)**

(OR)

(b) Let ψ be a function on (a, b). Then prove that ψ is convex on (a, b) if and only if for each x and y such that $a < x < y < b$, the graph of ψ on (a, x) and (y, b) does not lie below the line passing through $(x, \psi(x))$ and $(y, \psi(y))$. **(5)**

(c) (i) State and prove Holder's Inequality
(ii) State and prove Jensen's formula. **(7+8)**

(OR)

(d) State and Prove Minkowski's inequality. **(15)**

5. (a)Prove that the countable union of sets with respect to a signed measure v is a positive set. **(5)**

(OR)

(b) Let μ be a signed measure on $[\![X, S]\!]$ and let ϑ be a finite-valued signed measure on $[\![X, S]\!]$ such that $\vartheta \ll \mu$, then prove that given $\varepsilon > 0$ there exists $\delta > 0$ such that $|\vartheta|(E) < \epsilon$ whenever $|\mu|(E) < \delta$. **(5)**

(c) State and prove Jordan decomposition theorem. **(15)**

(OR)

(d)State and prove Radon-Nikodym Theorem. **(15)**

$$$$$$$$$