LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

SECONDSEMESTER – APRIL 2018

MT 2814- COMPLEX ANALYSIS

Date: 23-04-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answer all the questions.

1. a. Evaluate $\int_C \frac{sinz}{z} dz$where $C = e^{it}, 0 \leq t \leq 2\pi$.

OR

b. State and prove Morera's theorem. (5 marks)

c. If γ_0 and γ_1 are two closed rectiable curves in G and $\gamma_0 \sim \gamma_1$, then prove that $\int_{\gamma_0} f = \int_{\gamma_1} f$, for every function f analytic on G.

OR

d. State and prove Goursat's theorem. (15 marks)

2. a. State and prove Schwarz lemma.

OR

b. Define a convex function. Prove that a differentiable function f on [a,b] is convex if and only if f' is increasing. (5 marks)

c. State and prove the Riemann mapping theorem.

OR

d. State and prove Arzela-Ascoli theorem. (15 marks)

3. a. Let $Rez_n > 0, \forall n \geq 1$. Prove that $\prod_{n=1}^{\infty} z_n$ converges to a non zero number if and only if the series $\sum_{n=1}^{\infty} \log z_n$ converges.

OR

b. Prove that $\gamma(z+1) = z\gamma(z)$, for $z \neq 0, -1, ...$ (5 marks)

c. State and prove Bohr-Mollerup theorem.

OR

d. State and prove Mittag-Leffler's theorem. (15 marks)

4. a. State and prove Jensen's formula.

OR

b. Find the order of $exp(e^z)$and $\exp(z^n)$. (5 marks)

c. If f is an entire function of finite genus μ, then prove that f is of finite order $\lambda \leq \mu + 1$.

OR

d.State and prove Hadamard's factorization theorem. (15 marks)

5. a. Prove that the sum of residues of an elliptic function is zero.

OR

b. Prove that any two bases of the same module are connected by a unimodular transformation. (5 marks)

c. Prove the following:

1. $\varsigma'(z) = -\wp(z)$, where $\varsigma(z)$is weierstrass zeta functionand $\wp(z)$ is weierstrass $\wp$ function .
2. $\varsigma(z + w_1) = \varsigma(z) + n_1and\varsigma(z + w_2) = \varsigma(z) + n_2$ where n_1 and n_2 are constants.
3. $\sigma(z + w_1) = -\sigma(z)e^{n_1\left(z+\frac{w_1}{2}\right)}and\sigma(z + w_2) = -\sigma(z)e^{n_2\left(z+\frac{w_2}{2}\right)}$ where w_1 and w_2 are periods of Weierstrass $\wp$ function $\wp(z)$ and sigma function $\sigma(z)$.

OR

d. Obtain first order differential equation for Weierstrass$\wp$function $\wp(z)$.

(15 marks)

****