LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

SECONDSEMESTER – APRIL 2018

MT 2962- ACTUARIAL MATHEMATICS

Date: 27-04-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer all Questions. All questions carry equal marks.

I. (a) Define survival function $S(x)$.

OR

(b) Derive the time until death random variable for a person aged x. (5)

(c) If $S(x) = 1 - \frac{x^2}{100}, 0 \le x \le 10,$ then calculate (1) $F_X(x)$, (2) ${}_tp_4$, (3) ${}_{2/2}q_4$ and (4) probability density function of $T(4)$.

OR

(d) Prove that $\mu_x = \frac{f(x)}{s(x)}$ and $S(x) = \exp(-\int_0^x \mu_t dt)$.

II. (a) Given that $p_{40} = 0.999473$, calculate ${}_{0.4}q_{40.2}$ under the assumption of under distribution of death.

OR

(b) Explain uniform distribution of deaths and hence prove that $l_{x+t}= l_x - td_x$.

(c) For a certain population the probabilities q_x obtained for 5 weeks are $q_0 = 0.3, q_1 = 0.1, q_2 = 0.2, q_3 = 0.4, q_4 = 0.7$ and $q_5 = 1$ taking $l_0 = 100$. Construct the l, d_x, L_x, T_x, e_x.

OR

(d) What do you mean by a life table? With $S(x) = 1 - \frac{x}{100}, 0 \le x \le 100$

andl_0 =1,00,000, find l_1, l_4 and $l_{6.5}$. (15)

III. (a) Find the principle, if the amount with compound interest of 5% per annum is 3969 for the period of 2 years.

OR

(b) Give an account of endowment insurance policy. (5)

(c) (i) Find the amount of Rs10, 000/- after 10 years if the rate of interest is 5% and 5% per annum payable quarterly.

(ii) Give an account of whole life insurance policy. (10+5)

(d) An alumni association has 50 members, each of age *x*. It is assumed that all lives are independent. It is decided to contribute Rs. *R* to establish a fund to pay a death benefit of Rs .10, 000/- to each member. Benefits are to be payable at the moment of death. It is given that $\overline{A}_x = 0.06$ and ${}^2\overline{A}_x = 0.01$. Using normal approximation, find *R* so that with probability 0.95 the fund will be sufficient to pay the death benefit.

(15)

IV. (a) Derive the expression for present value of immediate annuity certain.

OR

(b) Explain the different classifications of annuities. (5)

(c) For a 3-year temporary life annuity-due on (30), given $S(x)=1-\frac{x}{80}, 0 \le x < 80$ $i=0.05$ and

$$Y=\begin{cases}\ddot{a}_{\overline{k+1}|}, k=0,1,2\\ \ddot{a}_{\overline{3}|}, \quad k=3,4,5\end{cases}$$, calculate *Var*(*Y*).

OR

(d) (i) Derive expression for $\ddot{a}_{\overline{n}|}$ and $\ddot{s}_{\overline{n}|}$.

(ii) Prove that $\ddot{a}_x = \frac{1-A_x}{d}$ (5+5+5)

V. (a) Define benefit premiums. A fully continuous 10 −year term insurance of face amount $Rs. 10{,}000/-$ has annual premium rate Rs. $100/-$ and the force of interest is 0.05. Find the value of the issue-date-loss if the death occurs exactly 5 years after issue.

OR

(b) On May 6, 1996, (67) bought a Rs. 1,00,000/- whole life insurance policy with death benefit payable at the end of the year of death. The policy is paid for by means of annual premiums, payable at the start of each year the policy remains in force. The policy holder died on August 6, 2003 and the loss to the insurer was Rs. 30,000/-. If *i*=0.06, what was the annual premium paid?

(5)

(c) (i) L is the loss-at-issue random variable for a fully discrete whole life insurance of 1 on (49). Calculate P and E(L) where $A_{49}=0.23882$, $\ddot{a}_{49}=13.4475$, ${}^2A_{49}=0.088873$, $i=0.06$ and $Var(L)=0.10$.

(ii) If ${}_{k|}q_x = c(0.96)^{k+1}, k=0,1,2,\ldots$ where c=0.04/0.96 and i=0.06, calculate P_x and Var(L).

(9+6)

OR

(d) (i) For (x), given the following information:

1) The premium for a 20-year endowment insurance of 1 is 0.0349.
2) The premium for a 20-year pure-endowment of 1 is 0.0230.
3) The premium for a 20-year deferred whole life annuity-due of 1 per year is 0.2087 and is paid for 20 years.
4) All premiums are fully discrete annual benefit premiums.
5) i=0.05.

Calculate the premium for a 20-payment whole life insurance of 1.

(ii) Prove that $\bar{P}(\bar{A}_x)=\frac{\bar{A}_x}{\bar{a}_x}$ and $Var(L)=\frac{{}^2\bar{A}_x-(\bar{A}_x)^2}{(\delta\bar{a}_x)^2}$. (9+6)

****