LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FOURTHSEMESTER – APRIL 2018

MT 4815- ADVANCED GRAPH THEORY

Date: 07-05-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answer ALL the questions. Each question carries equal marks.

1. a) Show that $\Gamma(G) = \Gamma(G^c)$ for a simple graph G. (5)

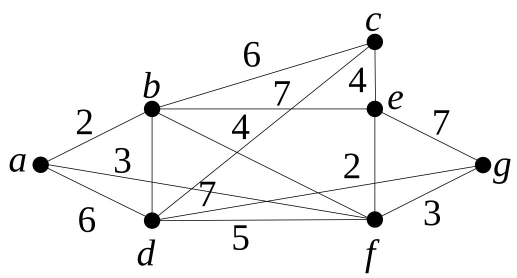
OR

b) Show that a sequence (d_1, d_2 … d_n) of non-negative integers is a degree sequence of some graph if and only if $\sum_{i=1}^{n} d_i$ is even. (5)

c) (i) Derive a characterization for bipartite graphs.
(ii) Show that if a k-regular bipartite graph with k> 0 has a bipartition (X, Y), then $|X| = |Y|$. (10 + 5)

OR

d) State Dijkstra's Algorithm. Use it, to determine the shortest path / distance between the vertex a and all other vertices of the following graph.



(15)

2. a) Prove that an edge e of a graph G is a cut edge of G if and only if e is contained in no cycle of G. (5)

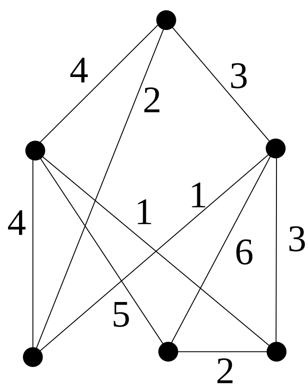
OR

b) Show that the closure of a graph G is well defined. (5)

c) (i) Find $\tau(K_n)$.
(ii) Derive the sufficient condition for a graph to be hamiltonian. (7 + 8)

OR

d) Explain Chinese Postman Problem. State Fleury's algorithm for Eulerian graphs. Obtain an optimal tour in the following weighted connected graph.



(15)

3. a) Prove that a matching M in a graph G is a maximum matching if and only if G contains no M-augmenting path. (5)

OR

b) If G is a bipartite graph, prove that $\chi' = \Delta$. (5)

c) State and prove Hall's theorem. (15)

OR

d) Derive the necessary and sufficient condition for a graph to have a perfect matching. (15)

4. a) Prove, with usual notation, that $\alpha' + \beta' = \nu$, if $\delta > 0$. (5)

OR

b) If G is a simple graph, prove that $\pi_k(G) = \pi_k(G - e) - \pi_k(G.e)$ for any edge e of G. (5)

c) State and prove Dirac's theorem. (15)

OR

d) State and prove Brook's theorem. (15)

5. a) Prove that K_5 is planar. (5)

OR

b) Let G be a nonplanar connected graph that contains no subdivision of K_5 or $K_{3, 3}$ and has as few edges as possible. Prove that G is simple and 3-connected. (5)

c) (i) State and prove Five-color theorem.

(ii) Find Euler's formula which relates the numbers of vertices, edges and faces in a connected plane graph. (9 +6)

OR

d) State and prove Kuratowski's theorem. (15)

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