LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTHSEMESTER – APRIL 2018

MT 5405- FLUID DYNAMICS

Date: 08-05-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

SECTION A

Answer **ALL** questions: (10 × 2 = 20)

1. Define stream lines.
2. Show that $\vec{q} = 2x\vec{\imath} - y\vec{\jmath} - z\vec{k}$ is a possible motion.
3. The velocity vector $\vec{q}$ is given by $\vec{q} = \vec{\imath}x - \vec{\jmath}y$, determine the equation of stream line.
4. Write down the boundary condition that the flow when it is at rest.
5. What is the complex potential of a source with strength m situated at the points $z = z_1$?
6. Find the stream function ψ, if $\varphi = A(x^2 - y^2)$ represents a possible fluid motion.
7. Find the vorticity components of a fluid motion, if the velocity components are $u = Ay^2 + By + C$, $v = 0$, $w = 0$.
8. Define vortex tube and vortex filament.
9. Find the vorticity vector for the velocity $\vec{q} = u\,\vec{\imath} + v\,\vec{\jmath}$
10. Define a two- dimensional sink and source.

SECTION B

Answer any **FIVE** questions: (5 × 8 = 40)

11. The velocity $\vec{q}$ in a 3-dimensional flow field for an incompressible fluid is $\vec{q} = 2x\vec{\imath} - y\vec{\jmath} - z\vec{k}$. Determine the equation of streamlines passing through the point (1, 1, 1).
12. Find the equation of streamlines and path lines of a flow given by $u = \frac{x}{1+t}, v = \frac{y}{1+t}, w = \frac{z}{1+t}$.
13. Derive the equation of continuity.
14. Prove that for the complex potential $\tan^{-1} z$ the streamlines and equipotentials are circles.
15. Obtain the complex potential due to the image of a source with respect to a plane.
16. Show that the velocity vector $\vec{q}$ is every where tangent to the lines in the XY-plane along which $\psi(x, y)$ = a constant.
17. Let $\vec{q} = (Az - By)\vec{\imath} + (Bx - Cz)\vec{\jmath} + (Cy - Ax)\vec{k}$, ($A$, B, C are constants) be the velocity vector of a fluid motion. Find the equation of vortex lines.
18. If the velocity of an incompressible fluid at the point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right)$ where $r^2 = x^2 + y^2 + z^2$. Prove that the fluid motion is possible and the velocity potential is $\frac{\cos\theta}{r^2}$.

SECTION C

Answer any **TWO** questions: (2 × 20 = 40)

19. (a).The velocity components for a two dimensional fluid system can be given in the Eulerian system by $u = 2x + 2y + 3t$, $v = x + y + \frac{t}{2}$. Find the displacement of a fluid particle in the Lagrangian system.
(b) Draw and explain the working of a Venturi tube. (12 + 8)

20. Derive the Euler's equation of motion and deduce the Bernoulli's equation of motion. (20)

21. (a)What arrangement of sources and sinks will give rise to the function $w = \log(z - \frac{a^2}{z})$?

(b)Obtain the complex potential due to the image of a source with respect to a circle. (12+8)

22. (a)Discuss the structure of an aerofoil.
(b)Derive Joukowski transformation (10 +10)

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