LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTHSEMESTER – APRIL 2018

MT 5406- COMBINATORICS

Date: 10-05-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Part A

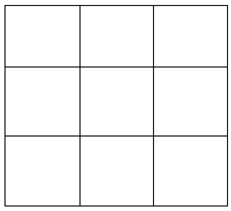
Answer ALL questions: (10 x 2 = 20)

1. Define falling factorial.
2. How many 7 letter words of binary digits are there?
3. In an examination a candidate has to pass in each of the five papers. How many different combinations of papers are there so that a student may fail?
4. Define Stirling number of second kind.
5. Define recurrence relation.
6. State generalized inclusion and exclusion principle.
7. Define permanent of a matrix.
8. Define derangement.
9. Find Euler's number for $n = 100$.
10. Define cycle index of a permutation group.

Part B

Answer any FIVE questions: (5 x 8 = 40)

11. There are 30 girls and 35 boys in a junior class while there are 25 girls and 20 boys in a senior class. In how many ways can a committee of 10 be chosen, so that there are exactly 5 girls and 3 juniors in the committee?
12. Prove that the number of distributions of n objects into m distinct boxes with the objects in each box arranged in a definite order is the rising factorial $[m]^n$.
13. State and prove multinomial theorem.
14. Derive the Pascals's identity using the concept of generating functions.
15. Derive the formula to find the sum of first n natural numbers using its recurrence formula given by $a_n - a_{n-1} = 1, n \geq 1$.
16. Determine the permanent of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.
17. Find the rook polynomial for the following chess board C.



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18. State and prove Sieve's formula.

Part C

Answer any TWO questions: (2 x 20 = 40)

19. (a) Derive the recurrence formula for S_n^m. Formulate a table for S_5^5.

(b) If there exists a bijection between the set of n-letter words with distinct letters out of an alphabet of m letters and the set of n-tuples on m letters without repetition, then show that the cardinality of each of these sets is $m(m-1)(m-2)\ldots(m-n+1)$. (10+10)

20. (a) In a town council there are 10 democrats and 11 republicans. There are 4 women among the democrats and 3 women among the republicans. Find the number of ways of planning committee of 8 councillors in a such a way that there are equal number of men and women and equal members from both parties.

(b) Show that the number of derangements of set with n objects is $D_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^n \frac{1}{n!}\right]$. (10+10)

21. State and prove ménage problem. (20)
22. State and prove Burnside's lemma. (20)

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