LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – NOVEMBER 2015

MT 1819 - PROBABILITY THEORY & STOCHASTIC PROCESSES

Date : 12/11/2015 Dept. No. Max. : 100 Marks

Time : 01:00-04:00

Section – A

Answer all the questions **10 x 2 = 20 marks**

1. If three fair coins are flipped , find the probability of getting at least one head.
2. Let an urn contain 7 white and 5 red marbles. If three marbles are chosen without replacement, find the probability of getting at least one red marble.
3. If two fair dice are rolled, find the probability of the sum to be 5 given that the sum is at least 3.
4. Define probability mass function of a random variable.
5. When binomial tends to Poisson distribution ?
6. Define marginal and conditional distributions.
7. Write any two properties of normal distribution.
8. Define convergence in probability.
9. Write the sufficient conditions for consistency of an estimator.
10. What is Markov property ?

Section-B

Answer any five questions **5x8= 40 marks**

11. State and prove addition theorem on probability for 'n' events.
12. If $f(x) = x^2/18$, $-3< x < 3$, zero elsewhere, find (i) $P(|X| < 1)$ (ii) $P(X^2 < 9)$.
13. Derive mean and variance of binomial distribution.
14. Let X_1,X_2,X_3 be a random sample from a distribution of the continuous type having the p.d.f. $f(x) = 2x$, $0 < x < 1$, zero elsewhere. Compute the probability that the smallest of these X_i exceeds the median of the distribution.
15. If $X_1,X_2,\ldots, X_n$ is a random sample from $N(0, \sigma^2)$, find the maximum likelihood estimator of σ^2.

16. Ten individuals are chosen at random from a normal population and their heights are found to be 63,63,66,67,68,69,70,70,71,71 inches. Test if the sample belongs to the population whose mean height is 66 inches. Use 5% significance level.

17. State and prove Boole's inequality.

18. If a Markov chain has the following transition probability matrix

$$P = \begin{Vmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \end{Vmatrix}$$

(i) Determine the classes and the periodicity of the states.

(ii) Check for the recurrence of the states.

Section-C

Answer any two questions **2 x 20 = 40 marks**

19.(a) State and prove Bayes' theorem. **(6 marks)**

(b) If $f(x) = (1/2)^x$, x = 1,2,3… , zero elsewhere, compute $\Pr(\mu - 2\sigma < X < \mu + 2\sigma)$. **(7marks)**

(c) Let X have the p.d.f. $f(x) = 2(1-x)$, $0 < x < 1$, zero elsewhere, find mean and variance**.** **(7 marks)**

20.(a) Derive the MGF of normal distribution. **(10 marks)**

(b) If X is N (75,100) , find (i) $\Pr(X < 60)$ (ii) $P(70 < X < 100)$ **(10 marks)**

21.(a) Let X_1 and X_2 have the joint p.d.f. $f(x_1,x_2) = x_1 + x_2$, $0< x_1< 1$, $0< x_2< 1$, zero elsewhere. Find the conditional mean and variance of X_2 given $X_1= x_1$, $0< x_1< 1$. **(12marks)**

(b) Derive mean and variance of beta distribution of first kind. **(8 marks)**

22.(a) State and prove central limit theorem. **(10 marks)**

(b) Derive $P_n(t)$ for Poisson process clearly stating the postulates. **(10 marks)**

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