# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **NOVEMBER 2015**

**MT 2502 - ALGEBRA AND CALCULUS - II**

Date : 12/09/2015 Dept. No. Max. : 100 Marks

Time : 09:00-12:00

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**PART- A**

Answer **ALL** Questions **(10 × 2 = 20)**

1.Evaluate $\int_0^1 x(1-x)^4 dx$.

2.Write the formulae to find the area of surface of revolution in Cartesian and polar co- ordinates.

3. Evaluate $\int_0^1 \int_0^2 xy^2\, dydx$.

4. If $u = x + y,\ y = uv$, then show that $\frac{\partial(x,y)}{\partial(u,v)} = u$.

5. Define Beta and Gamma functions.

6. Prove that $\Gamma(n+1) = n\Gamma(n)$ if $n > 0$.

7. If $u_1 + u_2 + \ldots + u_n + \ldots$ is convergent prove that $\lim_{n\to\alpha} u_n = 0$.

8. State D' Alembert's Ratio test.

9. Show that $\frac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots\ldots}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots\ldots} = \frac{e-1}{e+1}$.

10. If $y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$, show that $x = y + \frac{y^2}{2!} + \frac{y^3}{3!} + \frac{y^4}{4!} + \cdots$

**PART- B**

Answer any **FIVE**questions$(5\times 8 = 40)$

11. Show that $\int_0^{\pi/4} \log(1+\tan x)\, dx = \frac{\pi}{8}\log 2$.

12. A sphere is divided into two parts by a plane at a distance a/2 from the centre. Show that the ratio of the volumes of the two parts is 5 : 27.

13. Evaluate □ $x^2 dx\, dy$ over the area of the circle $x^2 + y^2 = a^2$.

14. Evaluate ( i) $\int_0^{\infty} e^{-x^2}\ dx$. (ii) $\int_0^{\pi/2} \sqrt{tanx}\ dx$

15. Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \frac{7}{4.5.6} + \ldots\ldots$

16. Show that the series $\sum \frac{\{(n+1)r\}^n}{n^{n+1}}$ is convergent if $r < 1$and divergent if $r \geq 1$.

17. Sum the series $1 + \frac{1}{3} + \frac{1.3}{3.\ 6} + \frac{1.3.\ 5}{3.\ 6.\ 9} + \ldots\ldots$

18. If $a, b, c$ denote three consecutive integers, prove that

$$\log b = \frac{1}{2} loga + \frac{1}{2} logc + \frac{1}{2ac+1} + \frac{1}{3}\cdot\frac{1}{(2ac+1)^3} + \ldots$$

**PART- C**

Answer any **TWO** questions (2× 20 = 40)

19.(a) If $I_n = \int sin^n x\ dx$ prove that $I_n = -\frac{cosx\ sin^{n-1}x}{n} + \frac{n-1}{n} I_{n-2}$.

Also evaluate $I_n = \int_0^{\pi/2} sin^n x\ dx$. (12)

(b) Find the surface area of the solid generated by rotating the cardioid

$r = a(1 + cos\theta)$about its line of symmetry. (8)

20. (a) Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\, dydx$ and evaluate it. (12)

(b) By changing into polar coordinates, find the value of the integral

$\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2)\, dydx$. (8)

21. (a)Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (10 )

(b) Find the limit of the sequence $\{a_n\}$ where $a_n = (1 + \frac{1}{n})^n$. (10)

22. (a).Sum the series $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!}$. (10)

(b) Show that $\frac{1}{1.3} + \frac{1}{2.5} + \frac{1}{3.7} + \ldots \ldots = 2 - 2log2.$ (10)

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