LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – November 2015

MT 5406 - COMBINATORICS

Date : 13/11/2015 Dept. No. Max. : 100 Marks
Time : 09:00-12:00

SECTION-A

ANSWER **ALL** THE QUESTIONS: **(10 x 2 = 20)**

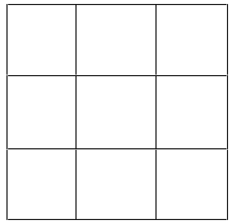
1. How many valid 8 digit binary numbers can be created? [Number should not start from zero.]
2. Define falling factorial.
3. Briefly explain recurrence relation with an example.
4. Find the sequences of the ordinary generating functions $3x^2 + e^{2x}$ and $(3 + x)^3$.
5. Define inclusion exclusion principle.
6. Find the co-efficient of x^5 in $(1 + x)(1 + 2x)(1 + 3x)(1 + 7x)(1 + 4x)$.
7. Define derrangement.
8. Define Euler's function.
9. Determine $\varphi(100)$.
10. Define cycle index of a permutation group.

SECTION-B

ANSWER ANY **FIVE** QUESTIONS: **(5 x 8 = 40)**

11. In a town council there are 10 democrats and 11 republicans. There are 4 women among democrats and 3 women among the republicans. Find the number of ways of forming a planning committee of 8 members which has equal number of men and women and equal number from both parties.
12. There are 5 Mathematics students and 7 Statistics students in a group. Find the number of ways of selecting 4 students from the group if
    a) there is no restriction.
    b) all 4 must be Mathematics students.
    c) all 4 must be Statistics students.
    d) all 4 must belong to the same subject.
13. Derive the Pascal's identity using the concept of generating functions.
14. Obtain the ordinary generating function (OGF) for the following sequences:
    a) (1,1,1,1,1,…)
    b) (1,-1,1,-1,1,…)
    c) (1,2,3,4,…)
    d) (1,0,1,0,1,0,…)
    e) (0,1,2,3,…)
15. State and prove Multinomial theorem .
16. Determine the permanent of the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

17. Find the rook polynomial for the given chess board *C*:



18. Derive the elements of the symmetries of a square?

SECTION-C

ANSWER ANY **TWO** QUESTIONS (2 x 20 = 40)

19. (i) Derive the recurrence formula for the Stirling number of first kind s_n^m . Formulate a table for s_5^5.
(ii) Prove that the number of distributions of n distinct objects into m distinct boxes with the objects in each box arranged in a definite order is the rising factorial $[m]^n$. (10 + 10)

20. (i) Derive the formula for the sum of the first n natural numbers using its recurrence formula given by $a_n - a_{n-1} = n, n \geq 1$.
(ii) Determine the OGF of the sequence $\{(r+n-1)C_{(n-1)}\}, r \geq 0$ by differentiation of infinite geometric series. (10 +10)

21. State and solve Ménage problem. (20)

22. (i) Determine the co-efficient of x^{27} in $(x^4 + x^5 + x^6 + \cdots)^5$.
(ii) State and prove Sieve's formula.
(8 +12)

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