



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION - MATHEMATICS

FIFTH SEMESTER – NOVEMBER 2015

MT 5505/MT 5501 - REAL ANALYSIS

Date : 03/11/2015
Time : 09:00-12:00

Dept. No.

Max. : 100 Marks

PART – A

ANSWER ALL QUESTIONS:

(10 x 2 = 20 marks)

1. Define similar sets and give an example.
2. Define a subsequence.
3. What is a discrete metric space?
4. Define open cover.
5. Define limit of a sequence.
6. Give an example of a continuous function which is not uniformly continuous.
7. When do you say that a function is strictly increasing?
8. Define local maximum and minimum of a function at a point.
9. State the linearity property of Riemann – Stieltjes integral.
10. Define absolute convergence of a sequence.

PART – B

ANSWER ANY FIVE QUESTIONS.

(5 x 8 = 40 marks)

11. Show that the number e is irrational.
12. Prove that the countable union of countable sets is countable.
13. Let (X, d) be a metric space. Prove that (i) union of an arbitrary collection of open sets in X is open in X , (ii) intersection of an arbitrary collection of closed sets in X is closed in X .
14. State and prove Intermediate value theorem for continuous functions.
15. State and prove Bolzano's theorem.
16. State and prove Lagrange's mean value theorem.
17. If f is monotonic on $[a, b]$, then the set of discontinuities of f is countable.
18. Show that the Riemann – Stieltjes integral is additive with respect to the interval of integration.

PART – C

ANSWER ANY TWO QUESTIONS.

(2 x 20 = 40 marks)

19. (a) State and prove Cauchy Schwarz inequality.
(b) Show that the set of all real numbers is uncountable.
20. Define a metric space and show that the Euclidean space R^n is a metric space.
21. Show that continuous image of a compact metric space is compact.
22. (a) State and prove Rolle's Theorem.
(b) State and prove Integration by parts rule.

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