LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2015**

MT 5508/MT 5502 - LINEAR ALGEBRA

Date : 11/09/2015 Dept. No. Max. : 100 Marks

Time : 09:00-12:00

PART – A

Answer ALL questions: **(10 x 2 = 20 marks)**

1. Define a vector space over a field F.
2. If V is a vector space over F, then prove that ao = o for $a \varepsilon F$.
3. Define a basis of a vector space.
4. Is the mapping $T: R^2 \rightarrow R^2$ defined by T (a,b) = (a+1, 2b, a+b) a homomorphism? Justify.
5. Define an inner product space.
6. Define Eigen values and Eigen vectors of a linear transformation.
7. For A, B ε F_n , prove that t_r (A+B) = t_r (A) + t_r(B).
8. If A and B are Hermitian, then prove that AB + BA is Hermitian.
9. Define an orthonormal set.
10. Show that $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is unitary.

PART – B

Answer any FIVE questions **(5 x 8 = 40 marks)**

11. Prove that the intersection of two subspaces of a vector space V is a subspace of V.
12. Prove that the vectors (1,0,0) , (1,1,0) and (1,1,1) form a basis of R^3 , where R is the field of real numbers.
13. Verify that $T: R^2 \rightarrow R^2$ defined by T(a,b) = (a – b, b – a, –a) for all a, b ε R is a vector space homomorphism. Find the rank and nullity of T.
14. Show that the vectors (1,1,) and (–3, 2) in R^2 are linearly independent to over R.
15. Prove that T ε A (V) is singular if and only if there exists an element N $\neq 0$ in V such that T(v) = 0.
16. Let T ε A (V) and λ ε F. Then prove that λ is an eigen value if T if and only of λ I – T is singular.
17. Show that any square matrix A can be expressed uniquely as the sum of a symmetric matrix and skew – symmetric matrix.
18. Prove that the eigen values of a unitary transformation are of absolute value 1.

PART – C

Answer any TWO questions **(2 x 20 = 40 marks)**

19. a) Prove that the vector space V over F is a direct sum of two of its subspaces w_1 and w_2 if and only if $V = w_1 + w_2$ and $w_1 \cap w_2 = (0)$.

    b) Let V be a vector space over R. If α, β, γ are linearly independent vectors of V, prove that the vectors $\alpha + \beta$, $\alpha - \beta$, $\alpha - 2\beta + \gamma$ are also linearly independent over R. (12+8)

20. a) If V is a vector space of dimension n and w is a subspace of V, then dim v/w = dim v – dim w.

    b) If A and B are subspaces of a vector space V over F, prove that $(A+B)/B \approx A/(A \cap B)$. (10+10)

21. a) Let $V = R^3$ and suppose that $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ is the matrix of T ε A (V) relative to the standard basis $N_1 = (1,0,0)$, $N_2 = (0, 1, 0)$, $v_3 = (0, 0, 1)$. Find the matrix of T relative to the basis $w_1 = (1, 1, 0)$, $w_2 = (1, 2, 0)$, $w_3 = (1, 2, 1)$.

    b) Let T be a linear transformation on R^2 defined by $T(a_1, a_2) = (2a_2, 3a, -a_2)$. Find the matrix of T relative to the standard basis $N_1 = (1, 0)$ and $N_2 = (0, 1)$. (12+8)

22. a) Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse.

    b) Show that the system of equations $\begin{matrix} x_1 + 2x_2 + x_3 = 11, \\ 4x_1 + 6x_2 + 5x_3 = 8, \\ 2x_1 + 2x_2 + 3x_3 = 19 \end{matrix}$ is inconsistent. (10 +10)

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