LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **NOVEMBER 2015**

MT 4503 - ALGEBRAIC STURUCTURE - I

Date : 11/09/2015 Dept. No. Max. : 100 Marks
Time : 01:00-04:00

PART – A

Answer ALL questions. **(10 ò 2 = 20)**

1. What is meant by an equivalence class of an equivalence relation on a set.
2. Give an example of a finite group.
3. Define order of an element of a group.
4. Show that every subgroup of an abelian group is normal.
5. Define homomorphism and epimorphism of a group.
6. Find the product of the permutations $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 5 & 6 & 3 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 6 & 2 & 1 & 5 \end{pmatrix}$.
7. In a ring R, show that for all $a \varepsilon R$, $a.0=0.a=0$.
8. Show that every field is an integral domain.
9. Define prime ideal of a ring.
10. Find all units of the ring Z(i).

PART – B

Answer any FIVE questions **(5 × 8 = 40)**

11. If H is a nonempty finite subset of a group G and H is closed under product in G, show that H is a subgroup of G.
12. Show that every group of prime order is cyclic.
13. Show that a subgroup N of a group G is normal in G if and only if the product of two left cosets is also a left coset.
14. If f is a homomorphism of a group G into a group G’, show that kernel f is a normal subgroup of G.
15. Define an alternating group of degree n and show that it is a normal subgroup of the symmetric group S_n.

16. Let R be a commutative ring with unit element whose only ideals are (0) and R. Show that R is a field.
17. Let R be a commutative ring with unit element and M be an ideal in R. Show that M is a maximal ideal if and only if R/M.
18. Let a,b be two non zero elements of a Euclidean ring R. If b is not a unit in R show that d(a)<d(ab).

PART – C

Answer any THREE questions. **(2 ò 20 =40)**

19(a) Show that union of two subgroups is a subgroup if and only if one is contained in the other.

(b) If G is a cyclic group of order n with generator a, show that a^m is also a generator of G if and only if m,n are relative prime. (10+10)

20 (a) State and prove fundamental theorem of group homomorphism.

(b) Let G be a group. Show that (i) the set of all inner automorphisms of G,I(G), is a normal subgroup of A(G)and (ii) I(G) ≈ G/Z(G) where Z(G) is the centre of G. (10+10)

21(a) Let R be a ring and A be an ideal of R. Show that R/A = { r+A : rε R } is also a ring.

(b) Show that an ideal of the ring Z of integers is a maximal ideal if and only if it is generated by a prime number. (10+10)

22(a) Show that Z(i) is an Euclidean ring.

(b) Show that every Euclidean ring is a principal ideal domain. (10+10)

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