

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034  
 B.Sc. DEGREE EXAMINATION – MATHEMATICS  
 FIRST SEMESTER – NOVEMBER 2002  
**MT 1500 / MAT 500 ALGEBRA, ANALYTICAL GEOMETRY,  
 CALCULUS AND TRIGONOMETRY**

09.11.2002  
 9.00 – 12.00

Max.: 100 Marks

**PART – A**

(10 × 2 = 20 Marks)

*Answer ALL questions. . Each question carries two marks.*

01. Find the slope of the curve  $r = a \sin 3\theta$  at pole.
02. Find the p– r equation of the cardioid  $r = a(1 - \cos \theta)$ .
03. Prove that in the curve  $x y = c^2$ , the subnormal varies as the cube of the ordinate.
04. Find the radius of curvature of the curve  $y = e^{3x}$  at the point where it crosses the y–axis.
05. Find the pole of  $3x + 8y - 24 = 0$  with respect to the ellipse  $9x^2 + 16y^2 = 144$ .
06. Find the equation of the chord of the parabola  $y^2 = 2x$  having (1,1) as its midpoint.
07. If  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 + px^2 + qx + r = 0$  find the value of  $\sum \alpha^2$ .
08. Solve  $2x^3 - 7x^2 + 4x + 3 = 0$  given  $1 + \sqrt{2}$  is a root.
09. Prove that the equation  $x^3 + 2x + 3 = 0$  has one negative root and two imaginary roots.
10. Prove that  $\tanh^{-1} x = \frac{1}{2} \log \frac{1+x}{e^{1-x}}$ .

**PART – B**

(5 × 8 = 40 Marks)

*Answer any FIVE questions. Each questions carries EIGHT marks.*

11. Prove that the spiral  $r = a\theta$  and the reciprocal spiral  $r = \frac{a}{\theta}$  intersect at right angles. (8)
12. Find the maxima and minima if any of the function  $f(x,y) = 12xy - 3y^2 - x^2$  subject to  $x + y = 16$ . (8)
13. Solve the equation  $x^4 + 2x^3 - 4x^2 - 22x + 40 = 0$  given that its root are in A.P (8)
14. Find the sum of the fourth powers of the roots of the equation  $x^3 - 2x^2 + x - 1 = 0$ . (8)
15. Solve the equation  $x^4 - 4x^2 + 8x + 35 = 0$  given that  $2 + i\sqrt{3}$  is a root. (8)

16. (i) If  $\tan \frac{x}{2} = \tanh \frac{x}{2}$ , show  $\cos x \cosh x = 1$
- (ii) Expand  $\cos 4\theta$  in terms of  $\sin \theta$ . (4+4)
17. (i) Expand  $\tan 7\theta$  in terms of  $\tan \theta$ .
- (ii) If  $\frac{\sin \theta}{\theta} = \frac{863}{864}$ , find an approximate value of  $\theta$ . (4+4)
18. Derive polar equation of a conic and prove that  $\frac{2}{\lambda} = \frac{1}{SP} + \frac{1}{SQ}$  where S is the focus of the conic. (4+4)

**PART – C**

(2 × 20 = 40)

Answer any TWO questions. Each question carries TWENTY marks.

19. (a) If  $y = (x + \sqrt{1+x^2})^m$  Prove that  $(1+x^2) y_{n+2} + (2n+1) x y_{n+1} + (n^2 - m^2) y_n = 0$ .
- (b) Find the evaluate of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . (10 +10)
20. (a) Define conjugate diameters of an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and if CP and CD are the semi conjugate diameters of an ellipse, find the locus of the point of intersection of tangents at P and D.
- (b) Prove that the product of perpendiculars from any point on the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  to its asymptotes is constant. (10+10)
21. a) Solve the equation  $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$ .
- (b) Find the real root of the equation  $x^3 + 6x - 2 = 0$  by Horner's method. (10+10)
22. (a) Separate into real and imaginary parts of  $\tanh (x+iy)$
- (b) Determine a,b,c such that  $\lim_{\theta \rightarrow 0} \frac{\theta (a + b \cos \theta) - C \sin \theta}{\theta^5} = 1$ .
- (c) Sum to n terms of the series  $\sin^2 \alpha + \sin^2 2\alpha + \sin^2 3\alpha + \dots \dots \infty$ . (6+6+8)

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