LOYOLA COLLEGE (AUTONOMOUS), CHENNAI –600 034.
M.Sc., DEGREE EXAMINATION – MATHEMATICS
I SEMESTER – NOVEMBER 2002
MT 1803/M 724 DIFFERENTIAL AND INTEGRAL EQUATIONS

13.11.2002
1.00 – 4.00 Max:100 marks

Answer all the questions.

I. a) i) State and Prove two useful formulae of ABEL.

ii) Show that the solutions e^x, e^{-x} and e^{2x} of $y''' - 2y'' - y' + 2y = 0$ are linearly independent and hence solve the given equation. (8)

b) i) Explain the method of variation of parameters.

ii) Solve $x'' - \frac{2}{t}x' + \frac{2}{t^2}x = t\sin t$, $x_1(t) = t$, $x_2(t) = t^2$ by the above method. (12+5)

iii) Suppose x_p is any particular solution of $L[x(t)] = d(t)$ where $t \in I$, $d(t)$ is continuous and if x_n is the general solution of the homogeneous, equation $L(x) = 0$, then prove that $x = x_p + x_n$ is the general solution of $L[x(t)] = d(t)$.

iv) Solve the homogeneous linear equation of second order with constant coefficients. (5+12)

II. a) i) Prove that $\left[J_{1/2}(x)\right]^2 + \left[J_{-1/2}(x)\right]^2 = \frac{2}{\pi x}$.

ii) Prove with usual that notation that $\frac{d}{dx}F(\alpha,\beta;\nu;x) = \frac{\alpha\beta}{\nu}F(\alpha+1,\beta+1,\nu+1,x)$ and deduce that $\frac{d^n}{dx^n}F(\alpha,\beta,\nu,x) = \frac{(\alpha)_n(\beta)_n}{(\nu)_n}F(\alpha+n,\beta+n,\nu+n,x)$. (8)

b) i) Solve the Bessel's equation $x^2y'' + xy' + (x^2 - n^2)y = 0$. (17)

(OR)

ii) State and prove Rodrigue's formula.

iii) Prove that $(1-2xz + z^2)^{-1/2} = \sum_{n=0}^{\infty} z^n P_n(x)$ where $|x| \le |, |z| < |$. (9+8)

III. a) i) Solve the homogeneous wave equation on a given set.

(OR)

ii) Convert $u_{xx} - 6u_{xy} + 9u_{yy} + u_x - e^{xy} - 1 = 0$ into its canonical form. (8)

b) i) Given a linear partial differential equation $Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu + G = 0$ where A, B, C, D, E, F are real constants, G is a real valued function of x, y defined on a set M and $A^2 + B^2 + C^2 \neq 0$ then if the above p.d.e is hyperbolic, Prove that for an affine transformation of the form $\bar{x} = \alpha_1 x + \beta_1, \bar{y} = \alpha_2 x + \beta_2 y$ the new coordinate system of p.d.e takes up the form $U_{\bar{x}\bar{y}} = \bar{D}u_{\bar{x}} + \bar{E}u_{\bar{y}} + \bar{F}u + \bar{G}$. Check whether the theorem is true for B = 0 and C = 0. (17)

(OR)

(ii) State and prove Cauchy's First Problem.

(iii) Find the solution of the Cauchy problem $u_{xx} - u_{yy} = 0$ with the condition

$$f(x,0) = x^2, \ \frac{\partial f(x,0)}{\partial y} = 4x3$$

(iv) Solve the boundary value problem of type – H where g(x) = 1-cos2x. (9+3+5)

IV. a) i) Solve $\varphi(x) = x + \int_0^x (S - x)\varphi(s)\, ds.$

ii) Prove that all iterated kernels of a symmetric Kernel are symmetric. (8)

b) i) Determine the eigen values and eigen functions of $\varphi(x) =$

$\lambda \int_{-1}^{1} (5xS^3 + 4x^2 S + 3xS)\varphi(s)\, ds$ and Prove that the eigen values of a symmetric Kernel are real. (10+7)

(OR)

ii) Show that a Volterra integral equation of Second Kind $\varphi(x) = F(x) +$

$\lambda \int_0^x k(x,s)\varphi(s)\, ds$ has one and only one solution.

iii) Find the resolvent kernel of the Kernel of the kernel $K(x_1 s) = 2 - (x\text{-}s)$. (10+7)

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