

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI –600 034.
M.Sc., DEGREE EXAMINATION – MATHEMATICS
I SEMESTER – NOVEMBER 2002
MT 1900/M 775 MATHEMATICAL METHODS – I

13.11.2002
1.00 – 4.00

Max: 100 marks

Answer all the questions.

I. (a) If $A = \{1, 4\}$, $B = \{4, 5\}$, $C = \{5, 7\}$ Find $(A \times B) \cap (A \times C)$. (3)

(b) i) Verify $\overline{A \cup B} = \overline{A} \cap \overline{B}$ using Venn diagram.

(OR)

ii) In a survey of 5000 persons, it was found that 2,800 read Indian Express and 2,300 read Statesman while 400 read both papers. How many read neither Indian Express nor Statesman? (7)

(c) i) Out of 880 boys in Loyola College, 224 play cricket, 240 play hockey and 336 play Basketball; of the total 64 play both basketball and hockey; 80 play cricket and basketball and 40 play cricket and hockey. 24 play all the three games. How many
(A) play only cricket, play only hockey, play only basketball
(B) play only one game
(C) play at least one game
(D) do not play any of the games. (10)

ii) Prove that $[A' \cup (A \cap B)'] = A \cap B$ by using law of sets. (5)

(OR)

iii) If $X = \{3, 6, 9, 12\}$ $Y = \{1, 2, 3, 4, 5, 6\}$ and $f : X \rightarrow Y$ is given by $f(x) = \frac{x}{3} + 1$.

Represent f as

(A) a set of ordered pairs

(B) an arrow diagram. Mention the type of function. (7)

iv) If $f: \mathbb{R} \rightarrow \mathbb{R}$, $g: \mathbb{R} \rightarrow \mathbb{R}$ and $h: \mathbb{R} \rightarrow \mathbb{R}$; $f(x) = 2x$, $g(x) = 3x-1$ and $h(x) = x^2+3$. Show that $(f \circ g) \circ h = f \circ (g \circ h)$. (8)

II. (a) Find the equation of the line passing through the points (3, 4) and (-5, 2). (3)

(b) (i) Find the point of intersection of the lines $3x - 4y + 6 = 0$ and $x - 2y - 3 = 0$

(OR)

(ii) Ten watches are sold when the price is \$ 80; 20 watches are sold when the price is \$ 60. What is the demand function. (7)

(c) (i) A manufacturer sells his product at \$ 5 per unit. Fixed cost are constant at \$ 3000, regard less of the number of units of products involved. Variable costs are estimated at 40 percent of total revenue. What is the total cost when 5000 units of product are sold. What is the break even point. (10)

(ii) Find the point of equilibrium for the following demand and supply equation.

$$Y = 10 - 2x$$

$$Y = \frac{3x}{2} + 1 \quad (5)$$

(OR)

(iii) Check whether the following lines are perpendicular: $x + y = 1$ and $x - y = 1$. (5)

(iv) When the price is \$ 25 no cameras of a fixed type are available for sale. For every \$10 increase in price 20 more cameras are available. What is the supply equation. (5)

(v) Suppose the fixed cost of production for a commodity is \$5000. The variable cost is \$ 7.50 per unit and the commodity sells for \$10 per unit what is the break – even point. (5)

III. a) Find $\frac{dy}{dx}$ when $Y = (4x + 3)^2$. (3)

b) (i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the function $x^2 + y^2 = 1$

[OR]

(ii) The demand function is given by $3x + 4y = 10$ where y is the price per unit and x is the number of units. Find the marginal revenue. (7)

c) (i) A radio manufacturer produces x sets per week at a total cost of RS. $\left(\frac{x^2}{25} + 3x + 100\right)$

He is a monopolist and the demand for his product is $x = 75 - 3p$ where p is the price in rupees per set. Show that the maximum net revenue is obtained when about 30 sets are produced per week. What is the monopoly price. (15)

(OR)

(ii) If a monopolist has total cost function $c = ax^2 + bx + C$ and if the demand law is

$P = \beta - \alpha x^2$ show that the output for maximum net revenue is $\frac{\sqrt{a^2 + 3\alpha(\beta - b)} - a}{3\alpha}$. (15)

IV. a) If $u = e^{x^2 + y^2}$ Find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$. (3)

b) (i) If $z = \frac{x + 4}{2x + 5y}$. Prove that $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$.

(OR)

(ii) Verify Euler's Theorem for $u(x, y) = x^3 - 2x^2 + 3xy^2 + y^3$. (7)

c) (i) The following are the demand functions for two commodities x_1 and x_2 : $x_1 = P_1^{-1.7} P_2^{0.8}$ and $x_2 = P_1^{0.5} P_2^{-0.2}$. Determine the four partial elasticities of demand.

(OR)

(ii) The demand for a quantity A is $q_1 = 100 - 10p_1 - 2p_2$.

A) Find the partial elasticity's $\frac{Eq_1}{Ep_1}$, $\frac{Eq_1}{Wp_2}$.

B) Find the partial elasticities for $P_1 = 3$, $P_2 = 5$.
