LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034.
M.Sc. DEGREE EXAMINATION – MATHEMATICS
SECOND SEMESTER – APRIL 2003

MT 2802 / M 823 – COMPLEX ANALYSIS – II

24.04.2003
1.00 – 4.00 Max : 100 Marks

Answer ALL questions.

I. (a) Show that $\sin \pi z = \pi z \prod_{n=1}^{\infty} (1 - \frac{z^2}{n^2})$. (8)

(b) Let Re Z_n >0 for all $n \geq 1$. Prove that $\prod_{n=1}^{\infty} z_n$ Converges to a non zero number if and only if the series $\sum_{n=1}^{\infty} \log z_n$ Converges. (8)

(c) (i) Obtain the Gauss formula for the Gamma function and show that $\Gamma(z+1) = z\Gamma z$ *for* $z \neq 0, -1, -2 \ldots$

(ii) If | z | ≤ | and p ≥ o then prove that $|1 - E_p(z)| \leq |z|^{p+1}$ (7+2+8)

OR

(d) (i) Let f be a real valued function defined on $(0, \infty)$, such that $f(x) > 0$ for all x > 0. Suppose f(x) statistics the following properties.

(1) log f (x) is convex,
(2) f (x+1) = xf (x) for all x,
(3) f (1) =1. Then show that f (x) = [(x) for all x.

(ii) State and prove Euler's Theorem.

II (a) State and prove first version of Maximum Principle for Harmonic Functions. (8)

OR

(b) Let $\gamma : [0,1] \to c$ be a path from a to b and let $\{(f t, D t) : 0 \leq t \leq 1\}$ be an analytic continuation along γ. There is a number ∈ >0 such that if $\sigma : [0,1] \to c$ is any path from a to b with $|\gamma(t) - \sigma(t)| < \varepsilon$ for all t, and if $\{(g t, \beta t) : 0 \leq t \leq 1\}$ is any contribution along σ with $[go]_a = [fo]_a$: than prove that $[g_1]_b = [f_1]_b$. (8)

(c) (i) Define Poisson kernal and prove the four properties of Poisson kernal.

(ii) Stating the required conditions, solve the Dirichlet's problem for the unit disk. (6+11)

OR

(d) State the prove Harnack's inequality and hence prove Hernack's theorem. (5+12)

III a. State and prove Poisson – Jenson formula. (8)

OR

b. State and prove Little Picard's theorem. (8)

c. Define order and genus of an entire function and prove that if f is an entire function of finite genus μ then f is of finite order $\lambda \leq +1$. (5+12)

OR

d. State and prove Bloch's theorem. (17)

IV a. Prove that a discrete module consists of ether of Zero alone, of the integral multiples nco of a single complex number $\neq$ o or 1 or of all linear combinations $n_1 w_1 + h_2 w_2$ with integral coefficient of two numbers w_1, w_2 with non real ration . (8)

OR

b. Show that the zeroes a_1, a_2 ….. a_n and poles b_1, b_2 ….b_n of an elliptic function satisfy (modm)

c. (i) Define weierslvass p function. Derive the differential equation satisfied by the weierslvass p –function.

(ii) Show that p (z +u) + p(z) + p (u) = (2+8+7)

OR

d. Show that !(z) is an odd function and prove that !(z) = -p(z), also derive the le gendre relation.

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