LOYOLA COLLEGE (AUTONOMOUS), CHENNAI –600 034.

B.Sc., DEGREE EXAMINATION – PHYSICS

III SEMESTER – NOVEMBER 2002

MT 3100/ MAT 100 MATHEMATICS FOR PHYSICS

08.11.2002

9.00 – 12.00 Max:100 marks

PART – A

Answer all questions. *(10 x 2 = 20 marks)*

01. Find L ($\cos^2 3t$).
02. Prove that $L[f'(t)] = S\,L[f(t)] - f(0)$.
03. Find the mean and standard deviation of normal distribution whose probability function is given by $f(x) = K\, e^{\frac{-1}{24}(x^2-6+9)}$ $(-\infty < x < \infty)$ where k is a constant.
04. In a Poisson distribution 3P(x = 2) = P (x = 4). Find the parameter λ.
05. Evaluate $\underset{x\to 0}{Lt}\left[\dfrac{a^2-1-x\log a}{x^2}\right]$.
06. Prove that the matrix $\begin{pmatrix}\cos\theta & \sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$ is Orthogonal.
07. Prove that the subtangent for any Point on the curve $y = k\, e^{x/a}$ is of constant length.
08. Find the lengths of the polar subtangent and polar subnormal for the curve $r = a\theta$.
09. Prove that $\underset{\theta\to 0}{Lt}\left[\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}\right] = 1$.
10. IF sin (A + iB) = x + iy Prove that $\dfrac{x^2}{\sin^2 A}-\dfrac{y^2}{\cos^2 A}=1$.

PART – B

Answer any Five questions. *(5 x 8 = 40 marks)*

11. Find L $\left[\dfrac{1-\cos t}{t}\right]$.

12. Find $L^{-1}\left[\dfrac{S^2-s+2}{S(S-3)(S+2)}\right]$.

13. If a, b, c denote three consecutive integers, show that

$$\log_e^b = \frac{1}{2}\log_e^a + \frac{1}{2}\log_e^c + \frac{1}{2ac+1} + \frac{1}{3}\cdot\frac{1}{(2ac+1)^3} + \ldots\ldots$$

14. Find the sum to infinity of the series $1+\dfrac{3}{4}+\dfrac{3.5}{4.8}+\dfrac{3.5.7}{4.8.12}+\ldots\ldots$

15. Find the rank of A = $\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix}$ if

a) a, b, & c are all different
b) two of a, b & c are equal and the 3rd is different
c) all a, b & c are equal .

16. A random variable X takes values 0, 1, 2, 3,….. with probability proportional to $(x+1)\left(\frac{1}{5}\right)^x$. Find a) $p(x \le 5)$ b) $p(x > 5)$

17. If $y = \sin(m \sin^{-1} x)$. Prove that $(1-x^2)y_2 - xy_1 + m^2y = 0$ and $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

18. If $\tan(x + iy) = u + iv$ Prove that $\frac{u}{v} = \frac{\sin 2x}{\sinh 2y}$.

PART – C

Answer any Two questions. *(2 x 20 = 40 marks)*

19. a) Sum the series $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!} + \dots to\ \infty$

b) Show that $\frac{5}{1.2.2.3} + \frac{7}{3.4.4.5} + \frac{9}{5.6.7} + \dots \dots \infty = 3\log 2 - 1$. (10+10)

20. a) Find the characteristic equation of the matrix A = $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & - \\ 1 & -1 & 2 \end{pmatrix}$ and Show that the matrix A satisfies Cayley – Hamilton theorem. Hence find A^{-1}.

b) Ten coins are tossed simultaneously. Find the probability of getting
i) at least 7 heads ii) exactly 7 heads iii) at most 7 heads (12+8)

21. a) Show that the rectangular hyperbolae $r^2 \sin 2\theta = a^2$ and $r^2 \cos 2\theta = b^2$ intersect at right angles.

b) Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is equal to $\frac{2a}{\sqrt{3}}$. (8+12)

22. a) Prove that $\cos^3\theta \sin^4\theta = \frac{1}{64}(\cos 7\theta - \cos 5\theta - 3\cos 3\theta + 3\cos\theta)$

b) Express $\frac{\sin 6\theta}{\sin\theta}$ in terms of $\cos\theta$. (10+10)

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