

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI –600 034.
B.Sc., DEGREE EXAMINATION – MATHEMATICS
III SEMESTER - NOVEMBER 2002

MT 3500/MAT 502 ALGEBRA, CALCULAS AND VECTOR ANALYSIS

06.11.2002

9.00 – 12.00

Max: 100 marks

PART – A

Answer all the questions.

(10 × 2 = 20 marks)

01. Define Beta and Gamma functions.
02. Solve. $pe^y = qe^x$.
03. Solve. $p = 2qx$.
04. Find the grad F of the function $F = 3x^2y - y^3z^2$.
05. State Gauss divergence Theorem.
06. Find L ($e^{5t} + 5 \cos t$)
07. Write the transformation from Cartesian to spherical Polar coordinates of a point.
08. State Wilson's Theorem. Is the converse true?
09. Find the remainder when 2^{1000} is divisible by 17.
10. Find $L^{-1} \left[\frac{1}{(S+a)^2} \right]$.

PART – B

Answer any Five questions.

(5 × 8 = 40 marks)

11. By changing the order of integration, evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy \, dx \, dy$.
12. Evaluate $\iint_R (x-y)^4 e^{x+y} \, dx \, dy$ where R is the square with vertices (1,0), (2,1), (1,2) and (0,1).
13. Solve. $Z = px + qy + \sqrt{1+p^2+q^2}$.
14. compute the divergence and curl of the vector $\vec{F} = xy^2 \vec{i} + 2x^2y \vec{j} - 3yz^2 \vec{k}$ at (1, -1, 1).
15. Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy \vec{i} - 5z \vec{j} + 10x \vec{k}$ along the Curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t=1$ to $t=2$.
16. a) Find L ($\sin^2 2t$) b) Find L ($t e^{-t} \sin t$) (3+5)
17. Find $L^{-1} \left(\frac{s-3}{s^2+4s+13} \right)$.
18. Find the highest power of 3 dividing 1000!.

PART - C

Answer any two questions.

(2 × 20 = 40 marks)

19. a) Prove that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma m + n}$.

b) Evaluate $\int_0^{\pi/2} \sqrt{\tan \theta} d\theta$.

c) Solve $x^2 p + y^2 q = (x+y)z$. (10+5+5)

20. a) Solve by Charpits method $p^2 + q^2 - 2px - 2qy + 1 = 0$.

b) Verify Green's Theorem in the plane for $\oint_C [(xy + y^2)dx + x^2 dy]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. (10 + 10)

21. a) Find $L \left[\frac{1 - e^t}{t} \right]$.

b) Using Laplace transform, solve $y'' + 2y' - 3y = \sin t$ given that $y(0) = 0, y'(0) = 0$. (5+15)

22. a) Show that $18! + 1$ is divisible by 437.

b) Show that if a, b, c are positive unequal quantities then $ax^{b-c} + bx^{c-a} + cx^{a-b} > a+b+c$.

c) State and Prove Fermat's Theorem. (5+5+10)
