LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – NOVEMBER 2002

MT 3801 / M 926 MECHANICS –I

07.11.2002 Max.: 100 Marks

1.00 – 4.00

Answer ALL the questions.

01. a) State and prove virtual work. (2+6)

(or)

b) A uniform sphere rolls down an inclined plane. Using Lagrangian Equation of motion, prove that the acceleration of the sphere is a constant. (8)

02. a) Derive Lagrangian equation of motion in the form

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right)-\frac{\partial L}{\partial q_i}=0 \; for, \; i=1 \; to \; n \qquad (17)$$

(or)

b) State and prove D'Alemterts principle and also find L for the motion of the Top. (2 + 8 +7)

03. a) Derive Hanilton's Canonical equations of motion. (8)

(or)

b) Find the Hamilton's function H for a spherical pendulum of length 'l' and hence deduce the equation of motion . (4+4)

04. a) State and prove the principle of Least Action. (17)

(or)

b) Define cyclic coordinate, give an example and derive Ruth's equation of motion for cyclic coordinates. Also find Ruth's function and equation of motion for the motion of a projectile. (17)

05. a) Discuss the canonical transformation whose generating function is ½ mw q^2 cot Q. (8)

(or)

b) Show that the transformation Q = long $(1+q^{½}\cos p)$, P = 2 $(1+q^{½}\cos p)\, q^{½}$ sin p is cannonical and find its generating function in the form $F_3 = -\left(e^{Q}-1\right)^2 \tan p$. (8)

06. a) State and prove JACOB1's identity. (17)

(or)

b) Show that the tansformation q = q (Q,P) and p = p(Q,P) and H = K will be cannonical i.e $\frac{\partial K}{\partial Q}=-\dot{P}$ and $\frac{\partial K}{\partial P}=\dot{Q}$ will be true only when $\frac{\partial(q,p)}{\partial(Q,P)}=1$. (17)

07. a) Derive Hamilton – Jacobi equation for Hamilton's principal function. (8)

(or)

b) Using the Hamilton–Jacobi equation for W discuss the motion of the Harmonic oscillator. (8)

08. a) For the Kepler problem find J_r , J_θ and J_ϕ. Hence find H. (17)

(or)

b) For S.H.M find J and solve the Hamilton Jacobi equation by separation of the variables for the motion of a particle in a plane under the action of a central force (17)

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