LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034.
M. Sc. DEGREE EXAMINATION – MATHEMATICS
IV SEMESTER – APRIL 2004
MT 4800/M 1025 – FUNCTIONAL ANALYSIS

Date :

Max. Marks : 100 Marks
Hours : 3 hours

Answer All questions:

1. a) Show that every vector space has a Hamel basis

(Or)

If $f \in X$, prove that the hull space Z(f) has deficiency O or 1 in X. Conversely, show that if Z is a subspace of X of deficiency O or 1, then there is an such that
Z = Z(f). (8)

b) (i) Let X and Y be normed linear spaces and let B(X, Y) denote the set of all bounded linear transformations from X into Y. Then prove that B(X, Y) is a normed linear space.

(ii) Let X and Y be normed linear spaces and let T : XY be a linear transformation. Prove that T is bounded if and only if T is continuous. (9 + 8)

(Or)

State and prove the Hahn – Banach Theorem (real version) (17)

2. a) Let X and Y be Banach spaces and let T be a linear transformation of X into Y. Prove that if the graph of T is closed, then T is bounded.

(Or)

State and prove F-Riesz Lemma (8)

b) State and prove the uniform boundedness theorem. Give an example to show that the uniform boundedness principle is not true for every normed vector space.

(Or)

If X and Y are Banach spaces and if T is a continuous linear transformation of X onto Y, then prove that T is an open mapping. (17)

3. a) State and prove Bessel's inequality

(Or)

If T is an operator on X, then show that *(Tx, x) = 0*

b) i) If x' is a bound linear functional on a Hilbert space X, prove that there is a unique such that $x'(x)$ = (x,

ii) If M and N are closed linear subspaces of a Hilbert space H and if P and Q are projections on M and N, then show that PQ = O QP = O. (9 + 8)

(Or)

Prove that two Hilbert spaces are isomorphic iff they have the same dimension. (17)

4) a) Define a topological divisor of Zero. Let S be the set of singular elements in a Banach algebra . Prove that the set of all topological divisors of Zero is a subset of S.

(Or)

Let A be a Banach algebra and Then prove that the spectrum of x, is non-empty.

b) State and prove the Spectral Theorem.

(Or)

Define the spectral radius of an element x in a Banach Algebra A. In the usual notation, prove that . (17)

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