LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

SECOND SEMESTER – NOVEMBER 2002

MT 2500/ MAT 501 ALGEBRA, ANALYTICAL GEOMETRY, CALCULUS II

02.11.2002 Max.: 100 Marks

1.00 – 4.00

PART – A (10 × 2 = 20 Marks)

Answer ALL questions, Each question carries TWO marks.

01. Evaluate $\int_0^{\pi/2} \log \tan x dx$.

02. State Bernoulli's formula.

03. Show that $1+\frac{1}{1!}+\frac{1}{2!}+.....$ is convergent.

04. State Canchy's root test for convergence of a given series.

05. Show that $\log_2 e - \log_4 e + \log_8 e - \log_{16} e ....\infty = 1$

06. Find the Coefficient of x^n in e^{a+bx}

07. Solve $(D^2-1)\ y = 0$

08. Evaluate $\int_0^{\pi/2} Sin^7 xdx.$

09. If a straight line makes and angle of 60° with each of the x and y axes, find the angle it makes with z axis.

10. Find the equation to the plane which passes through the point (–1,3,2) and is parallel to the plane x – y + z = 3.

PART – B (5 × 8 = 40 Marks)

Answer any FIVE questions. Each question carries EIGHT marks.

11. Find the surface area of the solid generated by revolution of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

12. If $I_n = \int_0^{\pi/2} x^n \sin x \ \ dx$ *and n is a positive* integer,

    show that $I_{n} + n(n-1)\ I_{n-2} = n\left(\frac{\pi}{2}\right)^n$.

13. If x is large show that $\sqrt{x^2+4} - \sqrt{x^2+1} = \frac{3}{2x} - \frac{15}{8x^3} + \frac{63}{16x^5}$ nearly.

14. Expand log $(1+x+x^2)$ in powers of x and show that the coefficient of x^n is either $-2/n$ or $1/n$ according as n is or not a multiple of 3.

15. (i) Test for convergence of the series $\sum_{n=o}^{\infty} \frac{n^3+1}{2^n+1}$

(ii) Test the convergence of $\sum \frac{1}{\sqrt{n^2+1}}$

16. Solve $(D^2+4)\, y = x \sin x$.

17. A variable plane which remains at a constant distance 3k from the origin and cuts the coordinate axes at A,B,C. Find the locus of the centroid of the triangle ABC.

18. Find the length of the perpendicular drawn from P(3, 4, 5) to the line $\frac{x-2}{2} = \frac{y-3}{5} = \frac{z-1}{3}$.

PART –C (20 × 2 = 40 Marks)

Answer any TWO questions. Each question carries TWENTY marks.

19. (i) If $I_n = \int_0^{\pi} \frac{\sin nx}{\sin x}\, dx$, show that $I_n = I_{n-2}$. Hence show that $I_n = 0$ or π according as n is even or odd.

(ii) Find the length of the arc of the parabola $y^2 = 4ax$ cut off by its latus rectum. (10 + 10)

20. Solve (i) $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$

(ii) $\frac{d^2y}{dx^2} + y$ = cosec x by variation of parameter method (8+12)

21. i) Show that the series $\frac{1}{1^K} + \frac{1}{2^K} + \frac{1}{3^K} + \ldots\ldots\ldots$ is convergent when K is greater than unity and divergent when k is equal to or less than unity

ii) Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \frac{7}{4.5.6} \ldots\ldots$ (12+8)

22. (i) Find the equation of the sphere which has its centre on the plane $5x + y - 4z + 3 = 0$ and passing through the circle $x^2 + y^2 + z^2 - 3x + 4y - 2z + 8 = 0$, $4x - 5y + 3z - 3 = 0$.

(ii) Find the shortest distance between the lines $\frac{x+3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ *and* $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$. Determine also its equation. (10+10)

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