LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – NOVEMBER 2002

MT 3800 / M 925 TOPOLOGY

05.11.2002 Max.: 100 Marks

1.00 – 4.00

Answer ALL questions.

01. a) (i) Let X be a metric space with metric d. Show that d_1 defined by

$$d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}$$ is also a metric on x.

Give an example of a pseudo – metric

(OR)

(ii) Let X be a metric space. Prove that any union of open sets in X is open and any finite intersection of open sets in X is open. (8)

b) i) Let X be a metric space, and let Y be a subspace of X. Prove that Y is complete iff it is closed.

ii) State and prove cantor's intersection theorem.

iii) If $\{A_n\}$ is a sequence of nowhere dense sets in a complete metric space X, prove that there exists a point in X which is not in any of the A_n's (6+6+5)

(OR)

(iv) Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous iff $f^{-1}(G)$ is open in X whenever G is open in Y.

(v) Let f: $X \to Y$ be a mapping of one topological space into another. Show that f is continuous $\Leftrightarrow f^{-1}(F)$ is closed in X whenever F is closed in Y $\Leftrightarrow f(\overline{A}) \subseteq \overline{f(A)}$ for every subset A of X. (8+9)

II (a) (i) If X is a second countable space, prove that X is separable

(OR)

(ii) If f and g are continuous real or complex functions defined on a topological space X, then prove that f + g, αf, and fg are also continuous. (8)

(b) (i) Show that any continuous image of a compact space is compact

(ii) Prove that any closed subspace of a compact space is compact.

(iii) Give an example to show that a compact subspace of a compact space need not be closed (6 + 6 + 5)

. (OR)

(iv) Prove that a topological space is compact if every subbasic open cover has a finite subcover. (17)

III (a) (i) Prove that a metric space is compact iff it is complete and totally bounded.

(OR)

(ii) Prove that every compact Hausdorff space in normal (8)

(b) (i) In a sequentially compact metric space, prove that every open cover has a Lebesque number.

(ii) Show that every sequentially compact metric space is totally bounded. (9+4+4)

(iii) Prove that every sequentially compact metric space is compact.

(OR)

(iv) If C(X,R) separates points, then show that X is C Hausdorff space. Prove that every completely regular space is Hausdosff space.

(v) Show that every compact subspace of a Hausdorff space is closed.

(vi) Show that a one–to–one continuous mapping of a compact space onto a Hausdorff space a homeomorphism (7+5+5)

IV (a) (i) Prove that the product of any non–empty class of connected space is connected.

(OR)

(ii) If $A_{1,} A_2$ …… A_n … is a sequence of connected subspaces of a topological space each of which intersects its successor, show that $\bigcup_{n=1}^{\infty}$ An is connected (8)

(b) (i) State and prove the URYSHON IMBEDDING THEOREM.

(OR)

(ii) State and prove the weierstrass Aproximation Theorem (17)

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