

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034
M. Sc. DEGREE EXAMINATION – PHYSICS
SECOND SEMESTER – NOVEMBER 2003
PH 2803 / PH 825 – MATHEMATICAL PHYSICS

15.11.2003
1.00 – 4.00

Max. : 100 Marks

PART – A

Answer ALL the questions.

(10 x 2 = 20)

01. Express $x^2 + y^2 = 25$ in zz^* and $re^{i\theta}$ representation.

02. State Liouville's theorem.

03. Find Laurent Series of $\frac{e^{2z}}{(z-1)^3}$ at $z = 1$ and name the Singularity.

04. Find the Jacobian of transformation of $w = z^2$.

05. Find $L(F_\epsilon(t))$, where $F_\epsilon(t)$ represents Dirac delta function.

06. State parseval's theorem.

07. Obtain the orthonormalizing constant for the set of functions given by $y = \sin \frac{n\pi x}{L}$; $n = 1, 2, 3 \dots$ in the interval $-L$ to $+L$.

08. Solve the differential equation $y' + k\lambda^2 y = 0$.

09. Write Laplace equation in spherical polar co-ordinates.

10. Using Rodrigue's formula for Legendre polynomials, evaluate $P_3(x)$.

PART – B

Answer any FOUR.

(4 x 7.5 = 30)

11. Derive the necessary conditions for a function to be analytic.

12. Find the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at its poles.

13. Expand $f(x) = \sin x$, $0 < x < \pi$ in a fourier cosine series and hence prove that

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \dots$$

(P.T.O)

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14. Verify that the system $y'' + \lambda y = 0$; $y'(0) = 0$ and $y(1) = 0$ is a Sturm-Liouville system. Find the eigenvalues and eigenfunctions of the system. Prove that eigenfunctions are orthogonal.

15. Prove that $L_{n+1}(x) = (2n + 1 - x)L_n(x) - n^2 L_{n-1}(x)$ where L's stand for Laguerre polynomials.

PART – C

Answer any FOUR.

(4 x 12.5 = 50)

16. (i) Evaluate $\int_{0,2}^{2,4} (2y + x^2) dx + (3x - y) dy$ along

a) the parabola $x = 2t$, $y = t^2 + 3$

b) straight lines from (0, 3) to (2,3) and then from (2,3) to (2,4) and

c) a straight line from (0, 3) to (2, 4). (7.5)

(ii) State and prove Poisson's Integral formula for a circle. (5)

17. Using contour Integration, evaluate $\int_0^{2\pi} \frac{d\theta}{a + b \sin \theta}$ for $a > |b|$.

18. An Inductor of 2 henrys, a resistor of 16 ohms and a capacitor of 0.02 farads are connected in series with an e.m.f E volts. Find the charge and current at any time $t > 0$ if a) $E = 300$ V and b) $E = 100 \sin 3t$ Volts

19. Generate Set of orthonormal functions from the sequence $1, x, x^2, x^3 \dots$ using Gram-Schmidt orthonormalization process.

20. Write Bessel's differential equation and obtain the standard solution.

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