

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI –600 034.

B.Sc., DEGREE EXAMINATION – MATHEMATICS

IV SEMESTER NOVEMBER 2002

STA 201 MATHEMATICAL STATISTICS

02.11.2002

9.00 – 12.00

Max:100 marks

SECTION – A

Answer All the questions.

(10 x 2 = 20 marks)

01. If $P(A) = P(B) = P(C) = \frac{1}{3}$, $P(AB) = P(BC) = P(AC) = \frac{1}{9}$ and $P(ABC) = \frac{1}{27}$, find the

probability that exactly zero event occur.

02. Define independent and mutually exclusive events.

03. If the p.d.f of the random variable X is $f(x) = \begin{cases} (1/2)^x; x=1,2,\dots \\ 0 & ; \text{elsewhere} \end{cases}$.

Find the mode.

04. State the properties of distribution function.

05. Let $f(x) = \begin{cases} \frac{1}{b-a}; a < x < b \\ 0 & , \text{elsewhere} \end{cases}$. Find E (X).

06. Define the conditional p.d.fs of Y given X and X given Y.

07. Define student's t distribution with 'γ' d.f.

08. Write the p.d.f of bivariate normal distribution.

09. Define consistent estimator.

10. State Neyman – Pearson lemma.

SECTION – B

Answer any Five questions.

(5 x 8 = 40 marks)

11. From the past experience with the illness of patients, a doctor has gathered the following information: 5% feel that they have cancer and do have cancer, 45% feel that they have cancer and don't have cancer, 10% do not feel that they have cancer and do have it and 40% feel that they do not have cancer and really do not have it. Denoting A be the event that the patient feels he has cancer and B be the event that the patient has cancer, find $P(A/B)$ and $P(B/A)$.

12. Let the d. F of a random variable X be $F(x) = \begin{cases} 0; x < 0 \\ x/8; 0 \leq x < 2 \\ x^2/16; 2 \leq x < 4 \\ 1 & ; x \geq 4 \end{cases}$.

Find E (X) and V (X).

13. Prevalence of diabetes in a community is one diabetic case in 10 persons. If 4 Children are born, what will be the probability of occurrence of diabetic in the following combinations.

i) All the 4 are normal

ii) At least 2 diabetic and

iii) 2 diabetic and 2 normal

14. Find the m.g.f of Poisson distribution and hence obtain its mean and variance.

15. If $X \sim N(12, 16)$, find i) $P(X \geq 20)$ ii) $P(0 < X < 12)$ and iii) Find x_1 if $P(X > x_1) = 0.24$.
16. If X and Y are independent Gamma variates with parameters μ and γ respectively, show that $\frac{X}{X+Y} \sim \beta(\mu, \gamma)$.
17. Show that the conditional mean of Y given X is linear in x in the case of bivariate normal distribution.
18. Derive the p.d.f of F – variate with (n_1, n_2) d.f.

SECTION – C

Answer any two questions.

(2 x 20 = 40 marks)

19. Let $f(x_1, x_2) = \begin{cases} 12x_1^2 x_2^3; & 0 < x_1 < x_2 < 1 \\ 0 & ; \text{elsewhere} \end{cases}$.

- Find i) the marginal distributions of X_1 and X_2 .
ii) correlation coefficient between X_1 and X_2 .

20. a) Find the mode of Poisson distribution
b) Find all odd and even order moments of Normal distribution.
21. a) X_1, X_2, \dots, X_n are independent random variables, each X_i having exponential distribution with parameter θ_i . Let $Z = \min(X_1, X_2, \dots, X_n)$. Show that Z has an exponential distribution with parameter $\sum_{i=1}^n \theta_i$.
b) Let (X, Y) have a bivariate normal distribution. Show that each of the marginal distributions is normal.
22. a) let X_1, X_2, \dots, X_n be a random sample of size 4 from $N(\theta, 1)$. Show that the sample mean is an unbiased estimator of the parameter θ .
b) Define i) Null and alternative Hypotheses
ii) Type I and type II error
iii) Critical region
iv) level of the test and
v) power of the test
