

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – NOVEMBER 2002

MT 3900 / M 955 METHODS OF OPERATIONS RESEARCH

09.11.2002

Max.: 100 Marks

1.00 – 4.00

Answer ALL the questions.

01. (i) What is the meaning of ‘fathoming’ a node as used in Branch and Bound algorithm? Under what conditions can a node be fathomed?
- (ii) Use Branch and Bound algorithm to solve following L.P.P
 Maximize $Z = 2x_1 + 2x_2$
 subject to $5x_1 + 3x_2 \leq 8$
 $x_1 + 2x_2 \leq 4$
 $x_1, x_2 \geq 0$, integer. (2 + 6)
- (OR)
- (b) Factorize a positive quantity b into n factors in such a way that the sum of their squares is a minimum using D.P technique. (8)
- (c) (i) Develop Gomory constraint for a mixed integer programming problem
 (ii) Max $Z = x_1 + x_2$
 such that $2x_1 + 5x_2 \leq 16$
 $6x_1 + 5x_2 \leq 30$,
 $x_1, x_2 \geq 0$ and x_1 , integer. (6 + 11)
- (d) An oil company has 8 units of money available for exploration of three sites. If oil is present at a site, the probability of finding it depends upon the amount allocated for exploiting the site, as given below.

Units of money allocated									
	0	1	2	3	4	5	6	7	8
Site 1	0.0	0.0	0.1	0.2	0.3	0.5	0.7	0.9	1.0
Site 2	0.0	0.1	0.2	0.3	0.4	0.6	0.7	0.8	1.0
Site 3	0.0	0.1	0.1	0.2	0.3	0.5	0.8	0.9	1.0

The probability that oil exists at the sites 1,2 and 3 is 0.4, 0.3 and 0.2 respectively. Find the optimal allocation of money by using D.P technique. (17)

02. a) State Economic lot size formula and show that $\frac{K}{k^*} = \frac{1}{2} \left[\frac{Q^*}{Q} + \frac{Q}{Q^*} \right]$ where Q^* is the optimum value of Q and K^* is the minimum cost under optimal policy. (3+5)

(OR)

b) Obtain Economic lot size with different rates of demand in different cycles. (8)

c) (i) Explain Selective Control techniques

(ii) Ten items kept in inventory by the school of Management studies at State University are listed below. Classify the items as A,B and C.

I tem	Annual usage	Valued unit (Rs.)
1	200	40.0
2	100	360.00
3	2,000	0.20
4	400	20.00
5	6,000	0.04
6	1,200	0.80
7	120	100.00
8	2,000	0.70
9	1,000	1.00
10	80	400.00

(5+12)

(OR)

d) Determine the optimum inventory level in a stochastic model which will minimize the expected cost stating the assumptions. (17)

03. a) A Branch of Punjab National Bank has only one typist. Since the typing work varies in length (number pages to be typed), the typing rate is randomly distributed approximating a Poisson distribution with mean service rate of 8 letters per hour. The letters arrive at a rate of 5 per hour during the entire 8 hour work day. If the type writer is valued at Rs.1.50 per hour, determine.

1. Equipment Utilisation
2. The percent time an arriving letter has to wait
3. Average system time
4. Average idle time cost of the typewriter per day

(OR)

b) A Project has the following time schedule.

Activity	Time in Month	Activity	Time in month
1-2	2	3-7	5
1-3	2	4-6	3
1-4	1	5-8	1
2-5	4	6-9	5
3-6	8	7-8	4
		8-9	3

- (i) Construct the network
(ii) Find the total float for each activity
(iii) Find the critical path and the project duration (8)
- (c) (i) Compare PERT and CPM
(ii) Consider a project consisting of nine jobs A,B,C D...I with the following precedence relations and time estimates

Job	Predecessors	Optimistic time 'a'	Most probable time 'm'	Pessimistic time 'b'
A	-	2	5	8
B	A	6	9	12
C	A	6	7	8
D	B, C	1	4	7
E	A	8	8	8
F	D,E	5	14	17
G	C	3	12	21
H	F,G	3	6	9
I	H	5	8	11

- (i) Calculate the expected length of the project.
(ii) The variance of the project duration
(iii) The standard deviation of the project duration (5+12)
(iv) What is the probability that the project will be completed within 50 days.
(v) What is the probability of completing the project 4 days sooner than expected.

(OR)

- d) (i) Use Dijkstra's algorithm to determine the shortest distance and the shortest path from node O to every other node in the network given below

- (ii) Consider the following directed network where s is the source node, n is the sink node, and the number along the arcs denote the capacities of flows.

Illustrate the following notions with an example for the above network.

- (i) A path connecting the source and the sink
- (ii) A cut separating the source and the sink
- (iii) The capacity of a cut.
- (iv) Find the maximum flow from the source to the sink using the labelling method
- (v) Find the minimum cut and verify the max-flow min-cut theorem. (8+9)

04. a) Consider the L.P.P

$$\text{Max } Z = 3x_1 + 5x_2$$

$$\text{subject to } x_1 \leq 4$$

$$3x_1 + 2x_2 \leq 18$$

$$x_1, x_2 \geq 0$$

(i) Solve this L.P.P

(ii) If a new variable x_5 is added to this problem with a column $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $C_5 = 7$ find the change in the optimal solution (8)

(OR)

(b) Solve the non-linear programming problem

$$\text{Minimize } Z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200.$$

$$\text{Subject to } x_1 + x_2 + x_3 = 1$$

$$x_j \geq 0, j = 1, 2, 3.$$

(c) Solve by Revised simplex method the following L.P.P

$$\text{Minimize } Z = -3x_1 + x_2 + x_3$$

$$\text{Subject to } x_1 - 2x_2 + x_3 \leq 11$$

$$-4x_1 + x_2 + 2x_3 \geq 3$$

$$2x_1 - x_3 = -1$$

$$x_1, x_2, x_3 \geq 0.$$

(17)

(OR)

d. Solve the following Quadratic programming by Wolf's method

$$\text{Maximize } Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

$$\text{Subject to } x_1 + 2x_2 \leq 2$$

$$x_1 + x_2 \geq 0.$$

(17)

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